Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

June 2013

Manitoba

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General Marking Instructions

Please make no marks in the student test booklets. If the booklets have marks in them, the marks need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of short-answer questions, long-answer questions, and multiple-choice questions. Short-answer questions are worth 1 or 2 marks each, long-answer questions are worth 3 to 5 marks each, and multiple-choice questions are worth 1 mark each. An answer key for the multiple-choice questions can be found at the beginning of the section "Booklet 2 Questions."

Each question is designed to elicit a well-defined response according to the associated specific learning outcome(s) and relevant mathematical processes. Their purpose is to determine whether a student meets the standards for the course as they relate to the knowledge and skills associated with the question.

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "0" and/or "NR" only (e.g., student was present but did not attempt any questions) please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

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Communication Errors

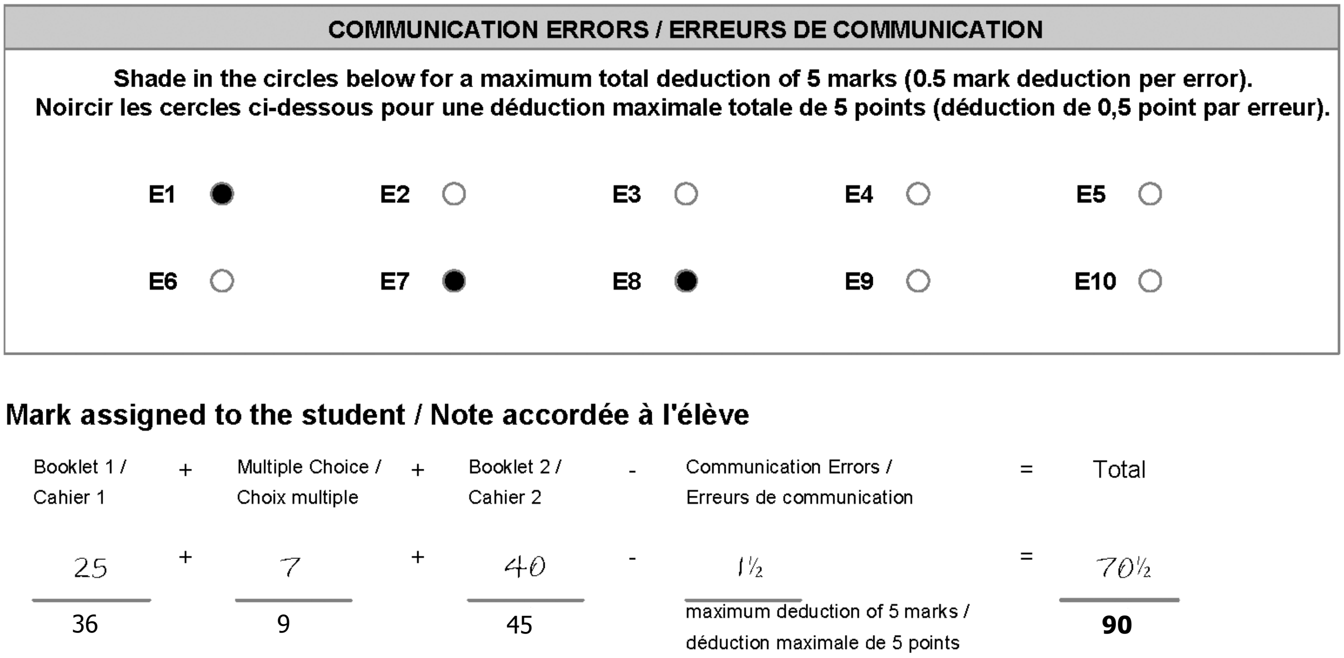
The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called "Communication Errors" (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e. committing a second error for any type will not further affect a student's mark), with a maximum deduction of 5 marks from the total test mark.

The total mark deduction for communication errors for any student response is not to exceed the marks given for that response. When multiple communication errors are made in a given response, any deductions are to be indicated in the order in which the errors occur in the response, without exceeding the given marks.

The student's final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.



COMMUNICATION ERRORS / ERREURS DE COMMUNICATION

Shade in the circles below for a maximum total deduction of 5 marks (0.5 mark deduction per error).
Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).

E1 ● **E2** ○ **E3** ○ **E4** ○ **E5** ○

E6 ○ **E7** ● **E8** ● **E9** ○ **E10** ○

Mark assigned to the student / Note accordée à l'élève

Booklet 1 / Cahier 1	+	Multiple Choice / Choix multiple	+	Booklet 2 / Cahier 2	-	Communication Errors / Erreurs de communication	=	Total
25	+	7	+	40	-	1½	=	70½
36		9		45		maximum deduction of 5 marks / déduction maximale de 5 points		**90**

Scoring Guidelines

Booklet 1 Questions

Question 1 T1

A central angle of a circle subtends an arc length of 5π cm.
Given the circle has a radius of 9 cm, find the measure of the central angle in degrees.

Solution

$$s = \theta r$$

$$5\pi = \theta(9)$$

½ mark for substitution into correct formula

$$\theta = \frac{5\pi}{9}$$

½ mark for solving for θ

$$\theta \text{ (in degrees)} = \frac{5\pi}{9} \bullet \frac{180^\circ}{\pi}$$

$$= 100^\circ$$

1 mark for conversion to degrees

2 marks

Solve the equation $\csc^2\theta + 3\csc\theta - 4 = 0$ over the interval $[0, 2\pi]$.
Express your answers as exact values or correct to 3 decimal places.

Solution

Method 1

$$\csc^2\theta + 3\csc\theta - 4 = 0$$

$$(\csc\theta - 1)(\csc\theta + 4) = 0$$

$\csc\theta = 1$	$\csc\theta = -4$	1 mark for solving for $\csc\theta$
$\sin\theta = 1$	$\sin\theta = -\frac{1}{4}$	1 mark for reciprocal of $\csc\theta$
$\theta = \frac{\pi}{2}$	$\theta_r = 0.252\ 680$	
or		
$\theta = 1.570\ 796$	$\theta = 3.394\ 273, 6.030\ 505$	

$\theta = \frac{\pi}{2}, 3.394, 6.031$

2 marks (1 mark for consistent solutions of each trigonometric equation)

or

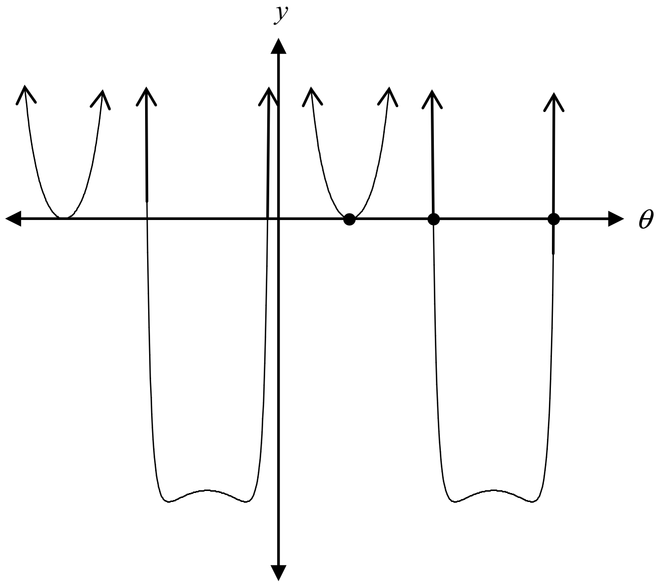
4 marks

$\theta = 1.571, 3.394, 6.031$

Method 2–Graphing Calculator

$$y = \left(\frac{1}{\sin\theta}\right)^2 + \frac{3}{\sin\theta} - 4$$

1 mark for equation



1 mark for justification

Find all zeros from $[0, 2\pi]$.

1 mark for restricted domain

$\theta = 1.571,\ 3.394,\ 6.031$

1 mark for solutions

4 marks

Question 3 R10

Jess invests \$12 000 at a rate of 4.75% compounded monthly.
How long will it take for Jess to triple her investment?

Express your answer in years, correct to 3 decimal places.

Solution

Method 1

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

$$36\ 000 = 12\ 000\left(1 + \frac{0.0475}{12}\right)^{12t}$$

½ mark for substitution

$$3 = \left(1 + \frac{0.0475}{12}\right)^{12t}$$

$$\ln 3 = \ln\left(1 + \frac{0.0475}{12}\right)^{12t}$$

½ mark for applying logarithms

$$\ln 3 = 12t\ln\left(1 + \frac{0.0475}{12}\right)$$

1 mark for power rule

$$t = \frac{\ln 3}{12\ln\left(1 + \frac{0.0475}{12}\right)}$$

½ mark for isolating *t*

$$t = 23.174\ 425$$

½ mark for evaluating quotient of logarithms

$$t = 23.174 \text{ years}$$

3 marks

Note(s):

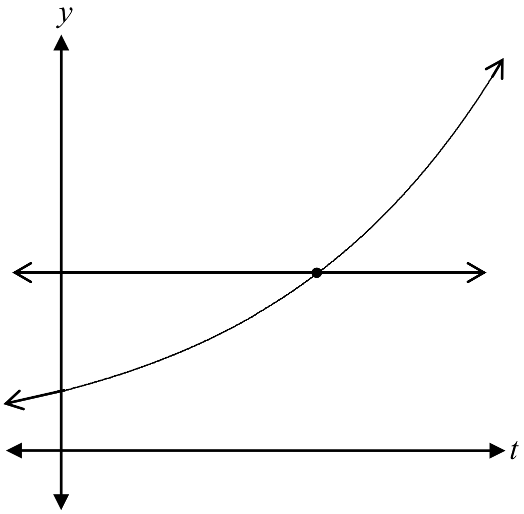
- award a maximum of 2 marks for the formula $A = Pe^{rt}$ used correctly

Method 2–Graphing Calculator

$y = 3$

$$y = \left(1 + \frac{0.0475}{12}\right)^{12t}$$

1 mark for equations



or

1 mark for justification

Find the value of t at the point of intersection of these two functions.

$t = 23.174$ years

1 mark for solution

3 marks

Question 4 P4

The 4th term in the binomial expansion of $\left(qx^2 - \frac{3}{x}\right)^{10}$ is $414\ 720x^{11}$.

Determine the value of q algebraically.

Solution

$t_4 = {}_{10}C_3\left(qx^2\right)^7\left(-\frac{3}{x}\right)^3$	2 marks (1 mark for ${}_{10}C_3$, ½ mark for each consistent factor)
$414\ 720x^{11} = 120\left(q^7x^{14}\right)\left(-\frac{27}{x^3}\right)$	½ mark for comparing coefficients
$414\ 720 = -3240q^7$	
$q^7 = -128$	
$q = -2$	½ mark for solving for q

3 marks

Question 5 P1

Bella has 2 pairs of shoes, 3 pairs of pants, and 10 shirts.
Carey has 4 pairs of shoes, 4 pairs of pants, and 4 shirts.
An outfit is made up of one pair of shoes, one pair of pants, and one shirt.

Who can make more outfits? Justify your answer.

Solution

Bella: $2 \times 3 \times 10 = 60$ outfits

Carey: $4 \times 4 \times 4 = 64$ outfits

$\therefore$ Carey can make more outfits.

1 mark for justification

1 mark

Question 6 P4

In the binomial expansion of $(x - y)^{10}$, how many terms will be positive?

Justify your answer.

Solution

Six terms will be positive.

1 mark for six terms

The term will be positive when "$-y$" has an even exponent.

1 mark for justification

2 marks

Question 7 T3, T5, T6

Solve the following equation algebraically where $180^\circ \le \theta \le 360^\circ$.

$$2\sin^2\theta + 5\cos\theta + 1 = 0$$

Solution

$2\left(1 - \cos^2\theta\right) + 5\cos\theta + 1 = 0$ 1 mark for identity

$2 - 2\cos^2\theta + 5\cos\theta + 1 = 0$

$2\cos^2\theta - 5\cos\theta - 3 = 0$

$\left(2\cos\theta + 1\right)\left(\cos\theta - 3\right) = 0$

$\cos\theta = -\frac{1}{2}$ $\cos\theta = 3$ 1 mark for solving for $\cos\theta$

$\theta_r = 60^\circ$ $\therefore$ no solution 1 mark for indicating no solution

$\theta = 240^\circ$ 1 mark for solving for θ

4 marks

Note(s):

- award a maximum of 3 marks if not solved algebraically

Solve the following equation algebraically:

$$\log_3(x-4)+\log_3(x-2)=1$$

Solution

Method 1

$$\log_3(x-4)+\log_3(x-2)=1$$

$$\log_3(x-4)(x-2)=1$$

1 mark for product rule

$$3^1=(x-4)(x-2)$$

1 mark for exponential form

$$3=x^2-6x+8$$

$$0=x^2-6x+5$$

$$0=(x-5)(x-1)$$

$$x=5 \qquad \cancel{x=1}$$

½ mark for solving for *x* within a quadratic equation
½ mark for rejecting extraneous root

3 marks

Method 2

$$\log_3(x-4)+\log_3(x-2)=1$$

$$\log_3(x-4)(x-2)=1$$

1 mark for product rule

$$\log_3\left(x^2-6x+8\right)=\log_3 3$$

½ mark for logarithmic form

$$x^2-6x+8=3$$

½ mark for equating arguments

$$x^2-6x+5=0$$

$$(x-1)(x-5)=0$$

$$\cancel{x=1} \qquad x=5$$

½ mark for solving for *x* within a quadratic equation
½ mark for rejecting extraneous roots

3 marks

Question 9 R1

Given that $f(x) = \{(1, 3), (2, 5), (3, 4), (4, 2)\}$, find $f(f(3))$.

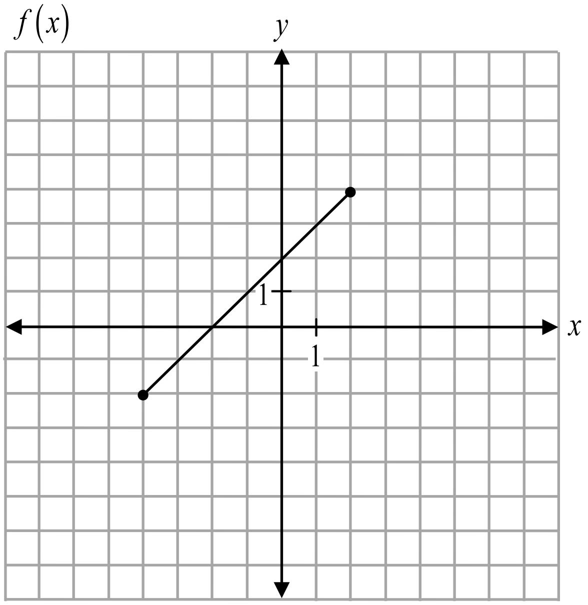
Solution

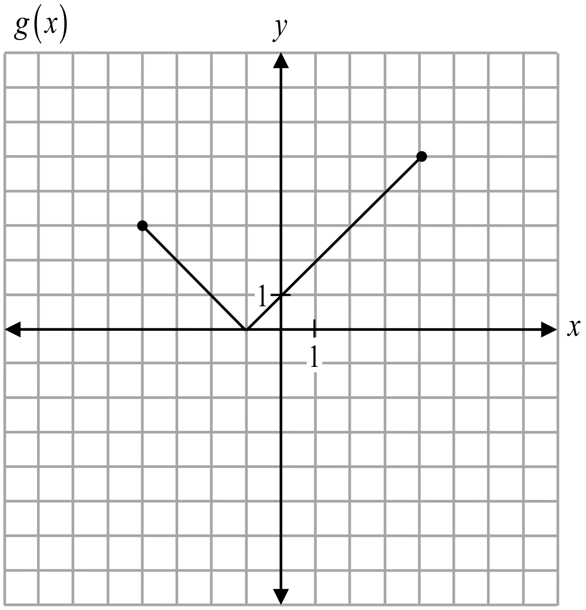
$f(f(3)) = f(4)$ ½ mark for $f(3) = 4$

$= 2$ ½ mark for $f(4) = 2$

1 mark

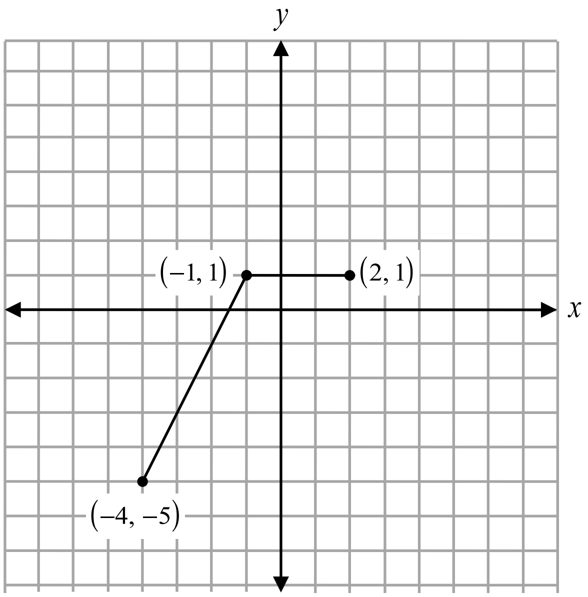
Given the graphs of $f(x)$ and $g(x)$ below,





sketch the graph of $y = f(x) - g(x)$.

Solution



x	$f(x)$	$g(x)$	$f(x)-g(x)$
-4	-2	3	-5
-2	0	1	-1
-1	1	0	1
0	2	1	1
2	4	3	1

1 mark for subtraction of $f(x)-g(x)$
1 mark for restricted domain

2 marks

Question 11 R2, R3

Given the graph of $y = f(x)$, describe the transformations to obtain the graph of the function $y = f(2x - 6)$.

Solution

Method 1

Factor out the 2.

$$y = f(2(x-3))$$

Horizontally compress by a factor of 2.
Then shift 3 units to the right.

1 mark for starting with a horizontal compression by a factor of 2
1 mark for ending with a horizontal shift of 3 units to the right

2 marks

Method 2

$$y = f(2x-6)$$

Shift 6 units to the right.
Then horizontally compress by a factor of 2.

1 mark for starting with a horizontal shift of 6 units to the right
1 mark for ending with a horizontal compression by a factor of 2

2 marks

Question 12 R5

Given $f(x) = \{(-3, 4), (2, 7), (8, 6)\}$, state the domain of the resulting function after $f(x)$ is reflected through the line $y = x$.

Solution

Domain: $\{4, 6, 7\}$

1 mark for correct domain

1 mark

Note(s):

- award ½ mark for stating the inverse of the function: $f^{-1}(x) = \{(4, -3), (7, 2), (6, 8)\}$

Question 13 R8

Determine the value of y in the following equation:

$$\log_x 27 - \log_x 3 = 2\log_x y$$

Solution

$\log_x 27 - \log_x 3 = 2\log_x y$

$\log_x \frac{27}{3} = 2\log_x y$ 1 mark for quotient rule

$\log_x 9 = \log_x y^2$ 1 mark for power rule

$9 = y^2$

$y = \pm 3$

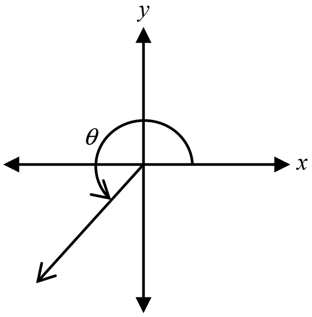
$y = 3 \qquad \xcancel{y = -3}$ ½ mark for positive value of y
½ mark for negative value of y and rejecting extraneous root

3 marks

Question 14 T1

Angle θ, measuring $\frac{5\pi}{4}$, is drawn in standard position as shown below.

Determine the measures of all angles in the interval $[-4\pi, 2\pi]$ that are coterminal with θ.



Solution

$\theta = -\frac{3\pi}{4}$ ½ mark

$\theta = -\frac{11\pi}{4}$ ½ mark

1 mark

Question 15 T6

Prove the identity below for all permissible values of x:

$$\frac{\sin^2 x}{\sec x + 1} = \cos x - \cos^2 x$$

Solution

Method 1

$$\begin{aligned}
\text{LHS} &= \frac{1 - \cos^2 x}{\frac{1}{\cos x} + 1} \\
&= \frac{1 - \cos^2 x}{\frac{1 + \cos x}{\cos x}} \\
&= \left(1 - \cos^2 x\right)\left(\frac{\cos x}{1 + \cos x}\right) \\
&= (1 - \cos x)(1 + \cos x)\left(\frac{\cos x}{1 + \cos x}\right) \\
&= (1 - \cos x)(\cos x) \\
&= \cos x - \cos^2 x \\
&= \text{RHS}
\end{aligned}$$

1 mark for correct substitution of identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Question 15 T6

Method 2

$$\text{LHS} = \frac{\sin^2 x}{\sec x + 1} \bullet \frac{(\sec x - 1)}{(\sec x - 1)}$$

$$= \frac{\sin^2 x(\sec x - 1)}{\sec^2 x - 1}$$

$$= \frac{\sin^2 x(\sec x - 1)}{\tan^2 x}$$

$$= \frac{\sin^2 x(\sec x - 1)}{\frac{\sin^2 x}{\cos^2 x}}$$

$$= \cos^2 x(\sec x - 1)$$

$$= \cos^2 x\left(\frac{1}{\cos x} - 1\right)$$

$$= \cos x - \cos^2 x$$

$$= \text{RHS}$$

1 mark for correct substitution of identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Question 16 P3

Solve algebraically:

$$ {}_{n}C_{2} = 4n + 5 $$

Solution

$$ {}_{n}C_{2} = 4n + 5 $$

$$ \frac{n!}{(n-2)!2!} = 4n + 5 $$ ½ mark for factorial notation

$$ \frac{n(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}\,2!} = 4n + 5 $$ ½ mark for factorial expansion
½ mark for simplification of factorial

$$ n(n-1) = 2!(4n+5) $$

$$ n^2 - n = 8n + 10 $$

$$ n^2 - 9n - 10 = 0 $$ ½ mark for simplification

$$ (n-10)(n+1) = 0 $$

$$ n = 10 \qquad \cancel{n = -1} $$ ½ mark for both values of *n*
½ mark for rejecting extraneous root

3 marks

Booklet 2 Questions

Answer Key for Multiple-Choice Questions

Question	Answer	Learning Outcome
17	C	P2
18	B	T1
19	A	P4
20	A	T4
21	D	T5
22	D	R4
23	B	R9
24	C	R12

Question 17 P2

How many different arrangements are possible when arranging all of the letters of the word SEPTEMBER?

a) $9!$

b) $6!3!$

[✓ c) $\frac{9!}{3!}$]

d) $\frac{6!}{3!}$

Question 18 T1

Which one of the following angles terminates in Quadrant III?

a) 3 radians

[✓ b) $\frac{7\pi}{5}$ radians]

c) -210°

d) 500°

Question 19 P4

There are 13 terms in the expansion of $\left(3x - y\right)^{2n}$. Determine the value of n.

[✓ a) 6]

b) 6.5

c) 7

d) 26

Question 20 T4

Which of the following is true about the periods of the three functions below?

$f(\theta) = 2\sin 3\left(\theta - \frac{\pi}{2}\right)$ $\quad g(\theta) = \sin 3\theta + 6$ $\quad k(\theta) = 3\sin\theta + 6$

a) The graphs of $f(\theta)$ and $g(\theta)$ have the same period.

b) The graphs of $g(\theta)$ and $k(\theta)$ have the same period.

c) All of the graphs have the same period.

d) None of the graphs have the same period.

Question 21 T5

Which of the following represents the general solution to the equation $\tan\theta = -1$?

a) $\theta = \frac{\pi}{4} + 2k\pi,\ k \in \mathrm{I}$

b) $\theta = \frac{\pi}{4} + k\pi,\ k \in \mathrm{I}$

c) $\theta = \frac{3\pi}{4} + 2k\pi,\ k \in \mathrm{I}$

d) $\theta = \frac{3\pi}{4} + k\pi,\ k \in \mathrm{I}$

Question 22 R4

If $(3, -2)$ is a point on the graph of $y = f(x)$, what point must be on the graph of $y = 2f(x+1)$?

a) $(4, -1)$ b) $(4, -4)$ c) $(2, 1)$ d) $(2, -4)$

Question 23 R9

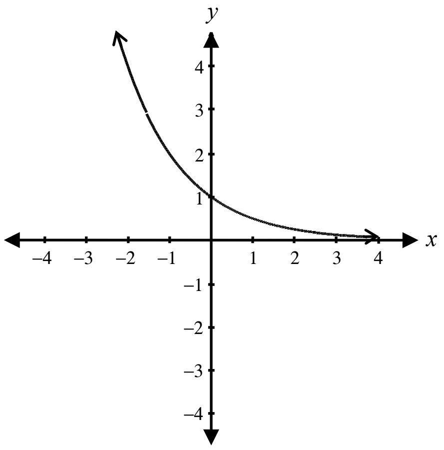
Which equation is represented by the graph sketched below?

a) $y = \left(\frac{1}{2}\right)^{-x}$

[✓ b) $y = \left(\frac{1}{2}\right)^{x}$]

c) $y = 2^x$

d) $y = -2^x$



Question 24 R12

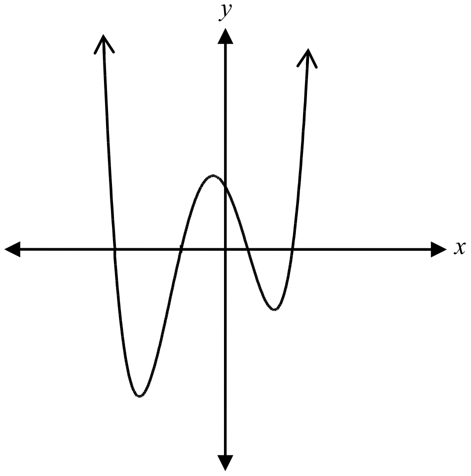
What is the degree of the polynomial represented below?

a) 2

b) 3

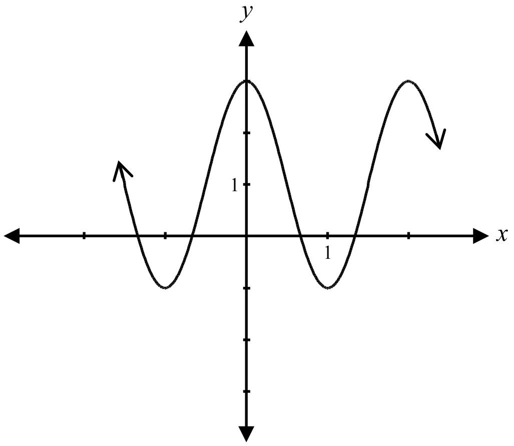
[✓ c) 4]

d) 5



Question 25 T4

Given the graph of $y = 2\cos\pi x + 1$ below, determine another equation that will produce the same graph.



Solution

Some sample equations are:

$$y = 2\cos\pi(x - 2) + 1$$

1 mark for correct equation

1 mark

$$y = -2\cos\pi(x - 1) + 1$$

$$y = -2\cos\pi(x + 1) + 1$$

$$y = 2\sin\pi\left(x + \frac{1}{2}\right) + 1$$

$$y = 2\sin\pi\left(x - \frac{3}{2}\right) + 1$$

Other answers are possible.

Question 26 R1

Given $f(x) = 3$ and $g(x) = x + 2$, determine the domain and range of $h(x) = \frac{f(x)}{g(x)}$.

Solution

Domain: $\{x \mid x \in \mathbb{R},\ x \neq -2\}$ 1 mark for domain

Range: $\{y \mid y \in \mathbb{R},\ y \neq 0\}$ 1 mark for range

2 marks

Question 27 T3

Explain how to find the exact value of $\sec\left(\frac{19\pi}{6}\right)$.

Solution

Find the exact value of $\cos\left(\frac{19\pi}{6}\right)$. 1 mark for $\cos\left(\frac{19\pi}{6}\right)$

Then take the reciprocal of the value of $\cos\left(\frac{19\pi}{6}\right)$. 1 mark for reciprocal

2 marks

Given $f(x) = 4 - x$, verify that $f^{-1}(x) = f(x)$.

Solution

Method 1

$$y = 4 - x$$

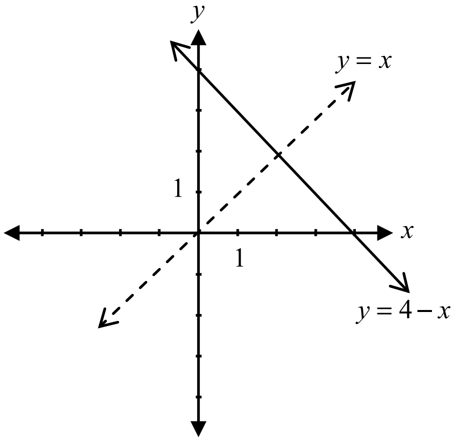
To find $f^{-1}(x)$, switch x and y values.

$$x = 4 - y$$
$$-y = x - 4$$
$$y = 4 - x$$

$f^{-1}(x) = 4 - x$ 1 mark for verifying $f^{-1}(x) = f(x)$

1 mark

Method 2



When $y = 4 - x$ is reflected over the line $y = x$ it produces the same graph. **1 mark**

Method 3

Assume $f^{-1}(x) = 4 - x$.

$$f\left(f^{-1}(x)\right) = 4 - (4 - x)$$
$$= x$$

$\therefore f(x)$ and $f^{-1}(x)$ are inverses of one another. **1 mark**

Sketch the graph of:

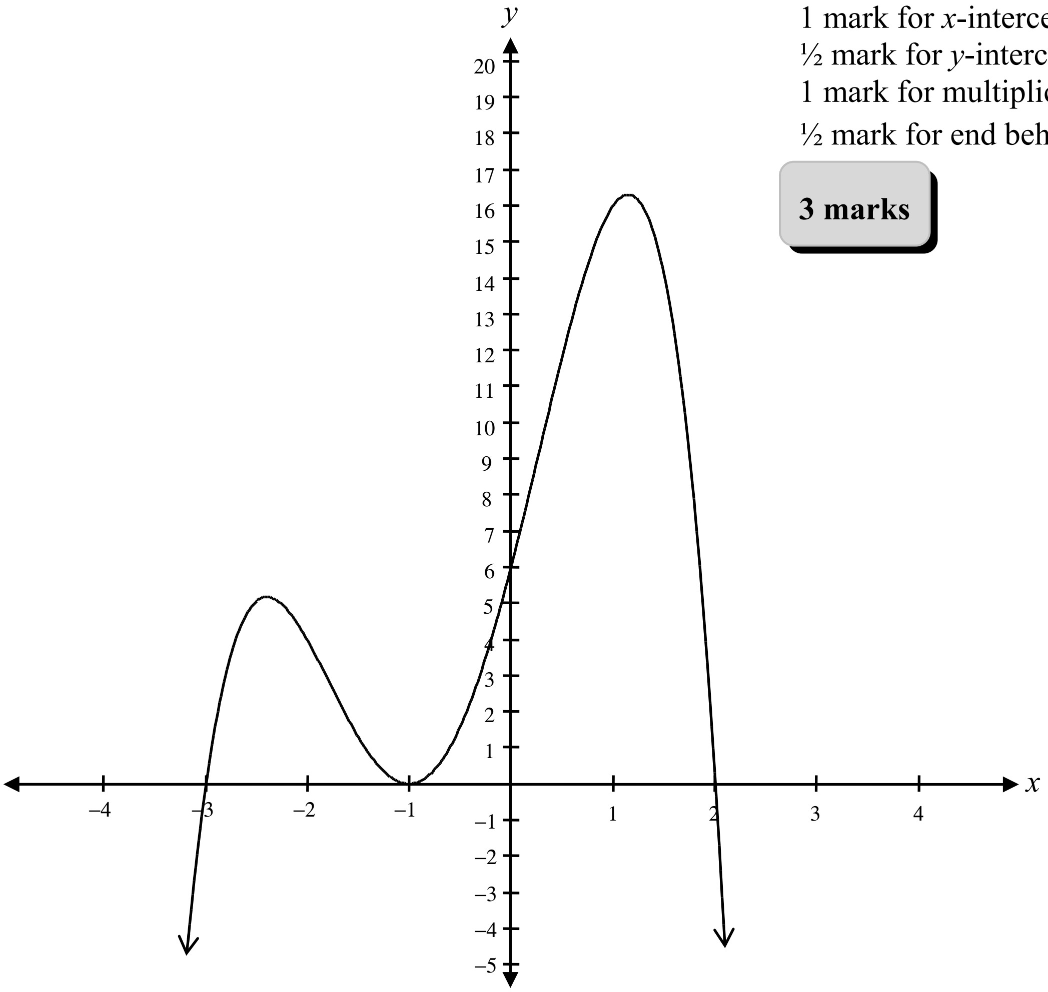
$$f(x) = (2 - x)(x + 3)(x + 1)^2$$

Label the x-intercepts and y-intercept.

Solution

x-intercepts: $-3, -1,$ and 2

y-intercept: 6



1 mark for x-intercepts
½ mark for y-intercept
1 mark for multiplicity of 2 at $x = -1$ only
½ mark for end behaviour

3 marks

Question 30 R7

Which expression has a larger value?

$\log_2 36$ or $\log_3 80$

Justify your answer.

Solution

Method 1

$\log_2 36$ $$2^? = 36 \begin{cases} 2^5 = 32 \\ 2^6 = 64 \end{cases} \approx 5.1$$

$\log_3 80$ $$3^? = 80 \begin{cases} 3^3 = 27 \\ 3^4 = 81 \end{cases} \approx 3.9$$

$\therefore \log_2 36$ is the larger value

1 mark for justification

1 mark

Method 2

$\log_2 32 = 5 \therefore \log_2 36$ is a little more than 5

$\log_3 81 = 4 \therefore \log_3 80$ is a little less than 4

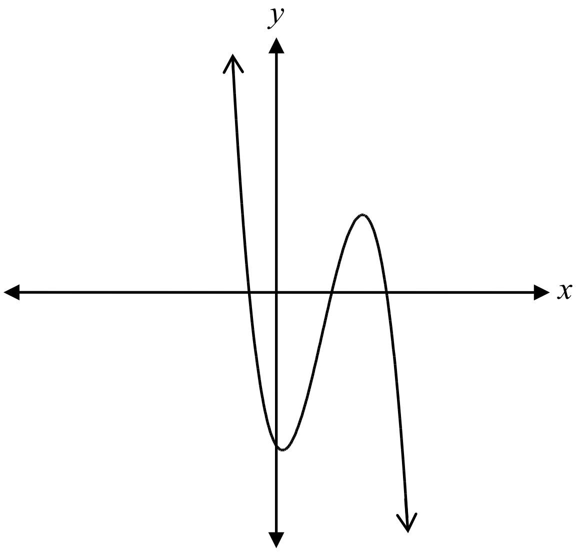
$\therefore \log_2 36$ is the larger value

1 mark for justification

1 mark

Question 31 R12

The graph below represents the equation $y = ax^3 + 6x^2 + 5x - 10$.



What must be true about the value of *a*? Explain your reasoning.

Solution

a is any negative number.	½ mark
Explanation with reference to end behaviour.	½ mark for explanation
	1 mark

or

a cannot be zero.	½ mark
The graph is of a cubic function, not a quadratic function.	½ mark for explanation
	1 mark

Question 32 T2

The terminal arm of an angle θ, in standard position, intersects the unit circle in Quadrant IV at a point $\mathrm{P}\left(\frac{\sqrt{5}}{4}, y\right)$. Determine the value of $\sin\theta$.

Solution

Method 1

The point $\mathrm{P}(\theta)$ on the unit circle has coordinates $(\cos\theta, \sin\theta)$.

$$\cos^2\theta + \sin^2\theta = 1$$

½ mark for showing $y = \sin\theta$

$$\left(\frac{\sqrt{5}}{4}\right)^2 + \sin^2\theta = 1$$

½ mark for substitution

$$\sin^2\theta = 1 - \frac{5}{16}$$

$$\sqrt{\sin^2\theta} = \sqrt{\frac{11}{16}}$$

$$\sin\theta = \pm\frac{\sqrt{11}}{4}$$

½ mark for solving for $\sin\theta$

$$\sin\theta = -\frac{\sqrt{11}}{4}$$

½ mark for a negative $\sin\theta$ value in Quadrant IV

2 marks

Method 2

$$\left(\sqrt{5}\right)^2 + y^2 = 4^2$$

½ mark for substitution

$$5 + y^2 = 16$$

$$y^2 = 11$$

$$y = \pm\sqrt{11}$$

½ mark for solving for y

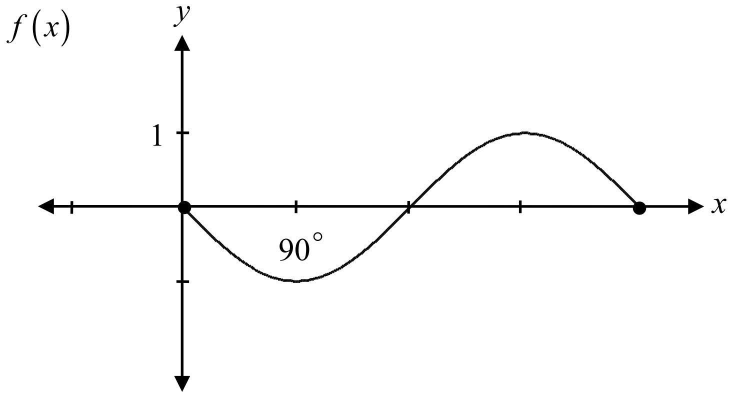
$$\sin\theta = -\frac{\sqrt{11}}{4}$$

½ mark for using the value of y to find the value of $\sin\theta$
½ mark for a negative $\sin\theta$ value in Quadrant IV

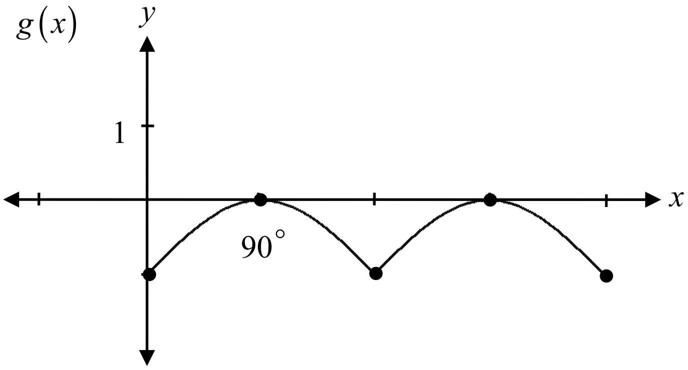
2 marks

Question 33 R1, R2

Given the sinusoidal function $f(x)$ below, sketch the graph of $g(x) = |f(x)| - 1$.



Solution



1 mark for absolute value
1 mark for vertical shift

2 marks

Question 34 R14

The graph of a rational function, $f(x)$, has a point of discontinuity when $x = 2$ and an asymptote when $x = 4$. Write a possible equation for $f(x)$.

Solution

A possible equation is:

$$f(x) = \frac{x - 2}{(x - 2)(x - 4)}$$

1 mark for $\frac{x - 2}{x - 2}$ (point of discontinuity when $x = 2$)

1 mark for $x - 4$ in denominator (asymptote when $x = 4$)

2 marks

Question 35 R11

Given that $(x - 1)$ is one of the factors, express $x^3 - 57x + 56$ as a product of factors.

Solution

$$\begin{array}{r|rrrr} 1 & 1 & 0 & -57 & 56 \\ & \downarrow & 1 & 1 & -56 \\ \hline & 1 & 1 & -56 & 0 \end{array}$$

$(x - 1)(x^2 + x - 56)$

or

$(x - 1)(x + 8)(x - 7)$

½ mark for $x = 1$

1 mark for synthetic division (or for any other equivalent strategy)

½ mark for consistent factors

2 marks

Question 36 T6

Give an example using values for A and B, in degrees or radians, to verify that $\cos(A + B) = \cos A + \cos B$ is **not** an identity.

Solution

Method 1

Let $A = 45^\circ$ and $B = 90^\circ$.

LHS	RHS
$\cos(45^\circ + 90^\circ)$	$\cos 45^\circ + \cos 90^\circ$
$\cos(135^\circ)$	$\cos 45^\circ + \cos 90^\circ$
$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2} + 0$
$-\frac{\sqrt{2}}{2}$	$\frac{\sqrt{2}}{2}$

1 mark for simplification of $\cos(A + B)$
1 mark for simplification of $\cos A + \cos B$

2 marks

LHS ≠ RHS $\therefore \cos(A + B) = \cos A + \cos B$ is not an identity.

Method 2

$$\cos(A+B)=\cos A\cos B-\sin A\sin B$$

Let $A=60^\circ$ and $B=30^\circ$.

$$\begin{aligned}\cos\left(60^\circ+30^\circ\right)&=\cos60^\circ\cos30^\circ-\sin60^\circ\sin30^\circ\\&=\left(\frac{1}{2}\right)\left(\frac{\sqrt{3}}{2}\right)-\left(\frac{\sqrt{3}}{2}\right)\left(\frac{1}{2}\right)\\&=\frac{\sqrt{3}-\sqrt{3}}{4}\\&=0\end{aligned}$$

1 mark for simplification of $\cos(A+B)$

$$\begin{aligned}\cos A+\cos B&=\cos60^\circ+\cos30^\circ\\&=\frac{1}{2}+\frac{\sqrt{3}}{2}\\&=\frac{1+\sqrt{3}}{2}\end{aligned}$$

1 mark for simplification of $\cos A+\cos B$

2 marks

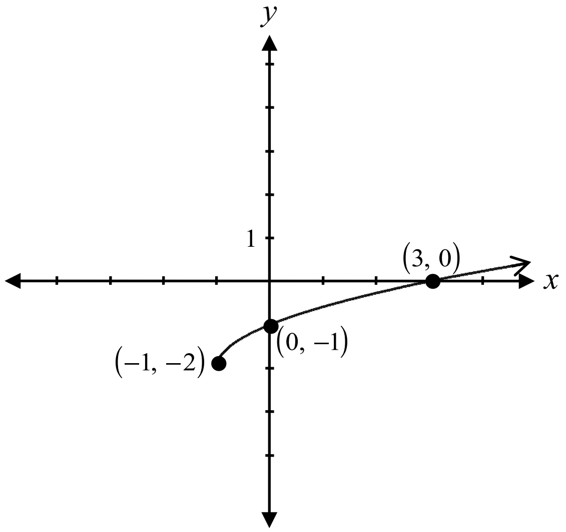
These two solutions are not equal $\therefore$ $\cos(A+B)=\cos A+\cos B$ is not an identity.

Question 37

R13

Sketch the graph of $y = \sqrt{x+1} - 2$ and verify that the value of the x-intercept is the same as the solution to the equation $\sqrt{x+1} - 2 = 0$.

Solution



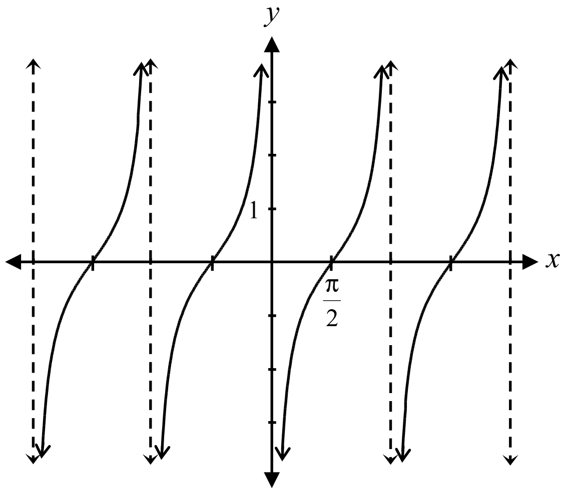
1 mark for general shape
½ mark for horizontal shift
½ mark for vertical shift

$$\begin{array}{rclcrcl}
\sqrt{x+1} & = & 2 & & \sqrt{x+1} - 2 & = & 0 \\
\left(\sqrt{x+1}\right)^2 & = & (2)^2 & \textbf{or} & \sqrt{3+1} - 2 & = & 0 \\
x + 1 & = & 4 & & \sqrt{4} - 2 & = & 0 \\
x & = & 3 & & 0 & = & 0
\end{array}$$

1 mark for verification

3 marks

Mohamed is asked to sketch the graph of $y = \tan x$.
His graph is shown below.



Explain why his graph is incorrect.

Solution

The graph of $y = \tan x$ should have zeros at $k\pi$, $k \in \mathrm{I}$.

1 mark for explanation

1 mark

or

The graph of $y = \tan x$ should have asymptotes at $(2k + 1)\frac{\pi}{2}$ or $\frac{\pi}{2} + k\pi$, $k \in \mathrm{I}$.

or

Mohamed sketched the incorrect graph. He sketched the graph of $y = \tan\left(x - \frac{\pi}{2}\right)$.

Question 39 T6

On the interval $0 \le \theta < 2\pi$, identify the non-permissible values of θ for the trigonometric identity:

$$\tan\theta = \frac{1}{\cot\theta}$$

Solution

$$\frac{\sin\theta}{\cos\theta} = \frac{1}{\frac{\cos\theta}{\sin\theta}}$$

$\therefore$ the above identity is non-permissible when $\cos\theta = 0$ or $\sin\theta = 0$.

1 mark for identifying non-permissible values (½ mark for $\cos\theta = 0$, ½ mark for $\sin\theta = 0$)

$\cos\theta \neq 0$

$\theta \neq \frac{\pi}{2}, \frac{3\pi}{2}$

$\sin\theta \neq 0$

$\theta \neq 0, \pi$

1 mark for solving for θ (½ mark for each solution set)

2 marks

$\theta \neq 0, \frac{\pi}{2}, \pi, \frac{3\pi}{2}$

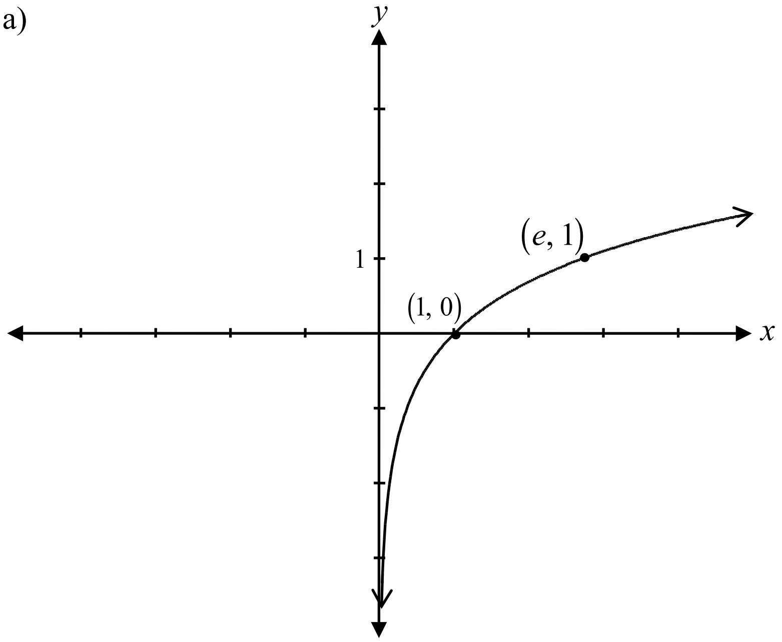
Question 40 a) R9 b) R2, R5

a) Sketch the graph of $y = \ln(x)$.

b) Sketch the graph of $y = -\ln(x - 2)$.

Solutions

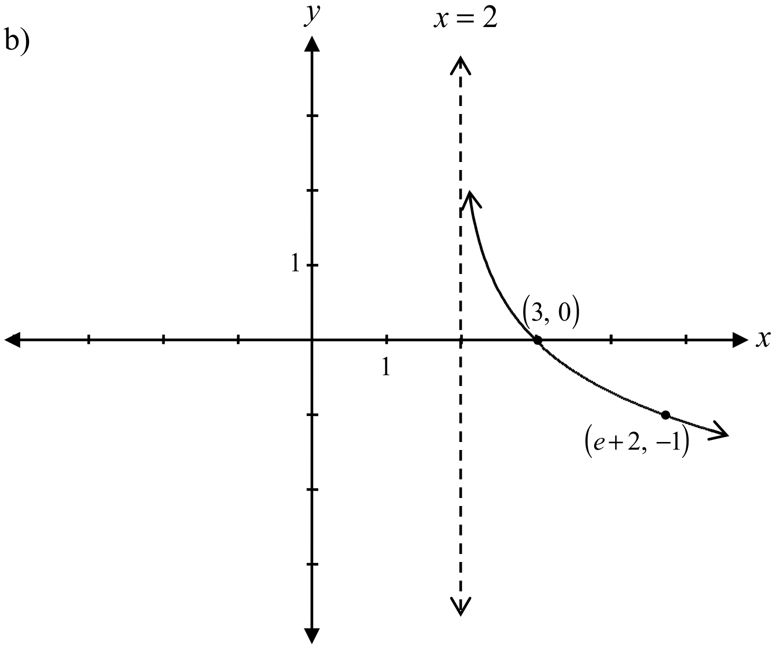
a)



½ mark for increasing logarithmic function
½ mark for x-intercept at $(1, 0)$
½ mark for consistent point on logarithmic function
½ mark for vertical asymptotic behaviour

2 marks

b)



1 mark for reflection in x-axis
1 mark for horizontal shift

2 marks

Question 41 R1, R13

Given $f(x) = \sqrt{x - 2}$ and $g(x) = 3x$, write the equation for $h(x) = f(g(x))$.

What are the restrictions on the domain of $h(x)$?
Explain your reasoning.

Solution

$h(x) = \sqrt{3x - 2}$

1 mark for $h(x) = f(g(x))$

$$\begin{aligned} 3x - 2 &\geq 0 \\ 3x &\geq 2 \\ x &\geq \frac{2}{3} \end{aligned}$$

½ mark for identifying restriction

Since we cannot find a square root of a negative number, there is a restriction on the domain, $x \geq \frac{2}{3}$.

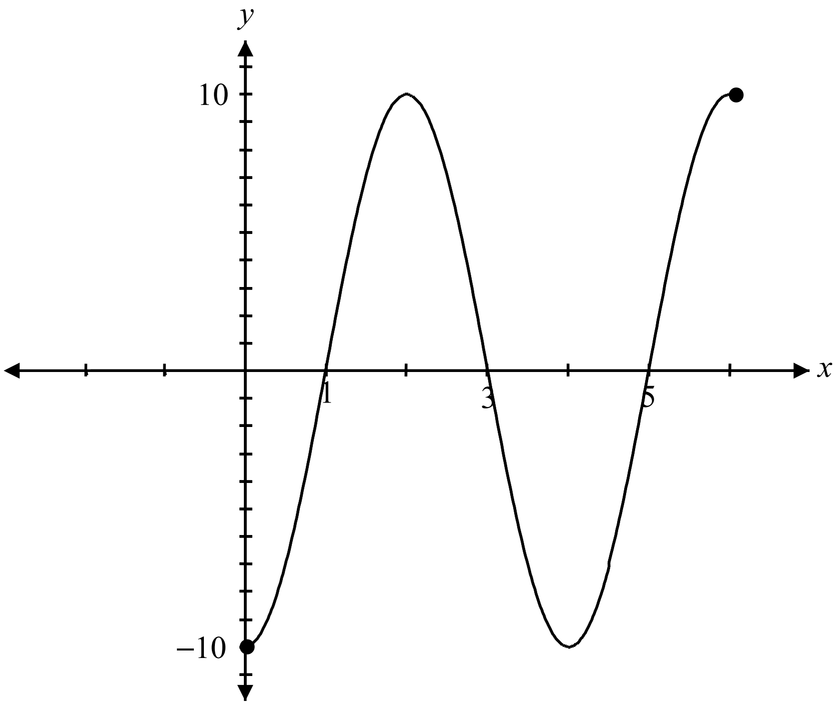
½ mark for explanation

2 marks

Sketch the graph of $y = 10\cos\left[\frac{\pi}{2}(x-2)\right]$ over the interval $[0, 6]$.

Solution

$$\text{period} = \frac{2\pi}{\frac{\pi}{2}} = 4$$



1 mark for amplitude
1 mark for period
1 mark for horizontal shift

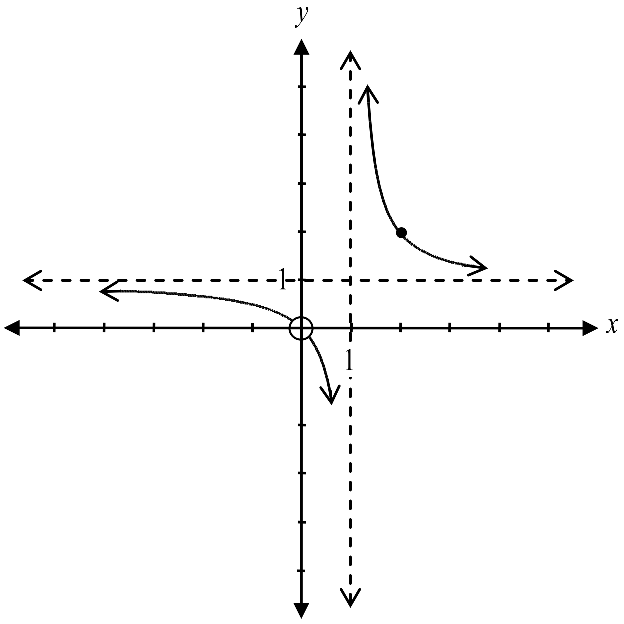
3 marks

Note(s):

- deduct ½ mark if the interval $[0, 6]$ is not completely sketched

Sketch the graph of the function $f(x) = \frac{x^2}{x^2 - x}$.

Solution



$$f(x) = \frac{x^2}{x(x-1)}$$

$$= \frac{x}{x-1}$$ with a point of discontinuity where $x = 0$

point of discontinuity: $f(0) = \frac{0}{0-1} = 0$

$\therefore$ there is a point of discontinuity at $(0, 0)$.

divide:

$$\begin{array}{r} 1 \\ x-1\overline{)\,x+0} \\ \underline{x-1} \\ 1 \end{array}$$

or

$$f(x) = \frac{x}{x-1} = \frac{x-1+1}{x-1} = 1 + \frac{1}{x-1}$$

$\therefore f(x) = \frac{1}{x-1} + 1$

$\therefore$ horizontal asymptote at $y = 1$

$\therefore$ vertical asymptote at $x = 1$

1 mark for vertical asymptote at $x = 1$
1 mark for horizontal asymptote at $y = 1$
1 mark for point of discontinuity at $(0, 0)$ or a point of discontinuity consistent with graph
½ mark for graph left of vertical asymptote
½ mark for graph right of vertical asymptote

4 marks

Question 44 R11

Is $(x-3)$ a factor of $x^4 - x^3 - 3x^2 + x - 1$?

Justify your answer.

Solution

Method 1

$x = 3$

½ mark for $x = 3$

$$\therefore (3)^4 - (3)^3 - 3(3)^2 + (3) - 1 = 81 - 27 - 27 + 3 - 1$$
$$= 29$$

1 mark for remainder theorem

The remainder does not equal zero,
therefore $(x-3)$ is not a factor.

½ mark for explanation

2 marks

Method 2

$$\begin{array}{l|ccccc} \underline{3} & 1 & -1 & -3 & 1 & -1 \\ & \downarrow & 3 & 6 & 9 & 30 \\ \hline & 1 & 2 & 3 & 10 & 29 \end{array}$$

½ mark for $x = 3$

1 mark for synthetic division

The remainder does not equal zero,
therefore $(x-3)$ is not a factor.

½ mark for explanation

2 marks

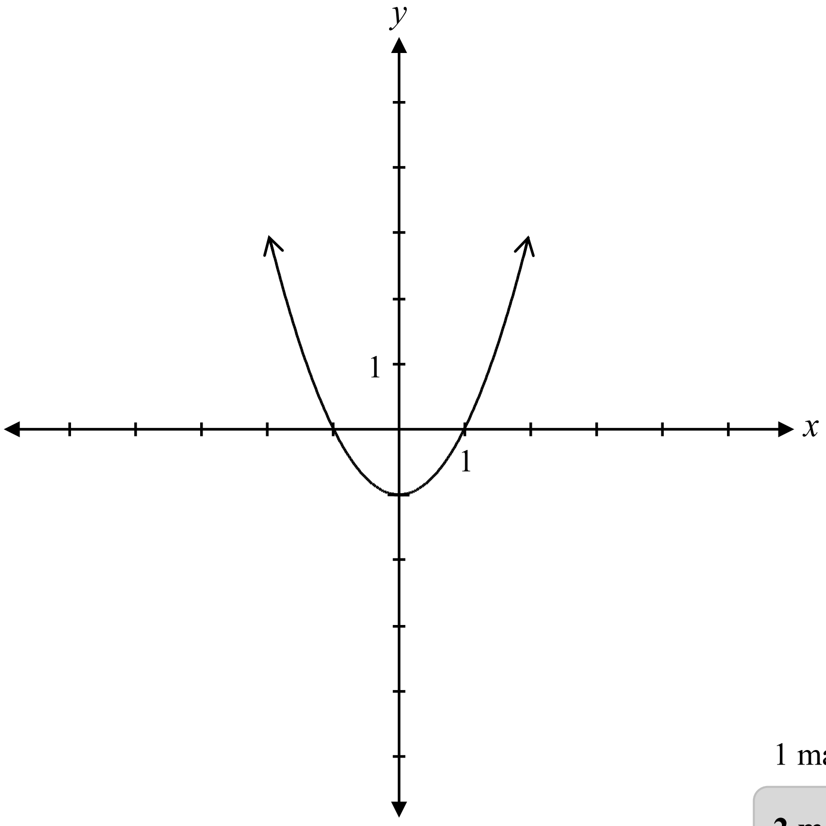
Given $f(x) = x - 1$ and $g(x) = x^2$, write the equation of $y = f(g(x))$ and sketch the graph.

Solution

$f(g(x)) = x^2 - 1$ 1 mark for composition

or

$y = x^2 - 1$



1 mark for consistent graph

2 marks

Appendices

Appendix A

MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error
- lack of clarity in explanation
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet*.

E1	▪ answer given as a complex fraction ▪ final answer not stated ▪ answer stated in degrees instead of radians or vice versa
E2	▪ changing an equation to an expression or vice versa ▪ equating the two sides when proving an identity
E3	▪ variable omitted in an equation or identity ▪ variables introduced without being defined
E4	▪ “$\sin x^2$” written instead of “$\sin^2 x$” ▪ missing brackets but still implied
E5	▪ missing units of measure ▪ incorrect units of measure
E6	▪ rounding error ▪ rounding too early
E7	▪ transcription error ▪ notation error
E8	▪ answer given outside the domain ▪ bracket error made when stating domain or range ▪ domain or range written in incorrect order
E9	▪ incorrect or missing endpoints or arrowheads ▪ scale values on axes not indicated ▪ coordinate points labelled incorrectly
E10	▪ asymptotes drawn as solid lines ▪ graph crosses or curls away from asymptotes

Appendix B

IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the Department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all “NR”) or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the Department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the Department along with the test materials.

Irregular Test Booklet Report

Test: ____________________

Date marked: ____________________

Booklet No.: ____________________

Problem(s) noted: ____________________

Question(s) affected: ____________________

Action taken or rationale for assigning marks: ____________________

Follow-up: ______________________________

Decision: ______________________________

Marker's Signature: ______________________________

Principal's Signature: ______________________________

For Department Use Only—After Marking Complete

Consultant: ______________________________

Date: ______________________________

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions

Question	Learning Outcome	Mark
9	R1	1
10	R1	2
11	R2, R3	2
12	R5	1
22	R4	1
26	R1	2
28	R6	1
33	R1, R2	2
40 b)	R2, R5	2
41	R1, R13	2
45	R1	2

Unit B: Trigonometric Functions

Question	Learning Outcome	Mark
1	T1	2
2	T3, T5	4
7	T3, T5, T6	4
14	T1	1
18	T1	1
20	T4	1
25	T4	1
27	T3	2
32	T2	2
38	T4	1
42	T4	3

Unit C: Binomial Theorem

Question	Learning Outcome	Mark
4	P4	3
5	P1	1
6	P4	2
16	P3	3
17	P2	1
19	P4	1

Unit D: Polynomial Functions

Question	Learning Outcome	Mark
24	R12	1
29	R12	3
31	R12	1
35	R11	2
44	R11	2

Unit E: Trigonometric Equations and Identities

Question	Learning Outcome	Mark
2	T3, T5	4
7	T3, T5, T6	4
15	T6	3
21	T5	1
36	T6	2
39	T6	2

Unit F: Exponents and Logarithms

Question	Learning Outcome	Mark
3	R10	3
8	R10	3
13	R8	3
23	R9	1
30	R7	1
40 a)	R9	2

Unit G: Radicals and Rationals

Question	Learning Outcome	Mark
34	R14	2
37	R13	3
41	R1, R13	2
43	R14	4