Grade 12
Applied Mathematics
Achievement Test

Student Booklet

January 2018

Manitoba

After the administration of this test, print copies of this resource will be
available for purchase from the Manitoba Learning Resource Centre.
Order online at www.manitobalrc.ca.

This resource will also be available on the Manitoba Education and
Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as
possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

GRADE 12 APPLIED MATHEMATICS
ACHIEVEMENT TEST

DESCRIPTION

Total Possible Marks: 63   **Time: 3 hours**

Unit	Marks
Relations and Functions	16
Probability	15
Design and Measurement	8
Financial Mathematics	17
Logical Reasoning	7

DIRECTIONS

Remember to

- indicate your input values by writing them in your booklet or printing a copy if using a technology tool

- express your answers in decimal and percentage form to the nearest hundredth (two decimal places) when rounding, unless otherwise indicated

  Example: $\dfrac{15}{29} = 0.52$ or 51.72%

- state any assumptions you make

A "graphic organizer" is a visual representation of information. Examples include a tree diagram, a chart, a list, a Venn diagram, a truth table, Pascal's triangle, etc.

A clearly communicated answer

— is easily identified in the response space

— includes the parameters in the equation, and "$y =$", "sin", "ln", or "x", as applicable

— includes the units of measure, where applicable

— includes labels, units, and scales for the axes on graphs

— is expressed as an exact value or is appropriately rounded

Marks may be deducted for errors relating to any of the above.

Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited.

RELATIONS AND FUNCTIONS

Question 1	**Total: 3 marks**

The temperature in an office is controlled by an electronic thermostat. The temperature varies according to the sinusoidal function:

$$y = 3 \sin(0.26x - 2.88) + 19$$

where y represents the temperature in degrees Celsius
and x represents the time in hours past midnight.

a) Determine the temperature in the office at 9 a.m.

(1 mark)

b) Determine the maximum temperature in the office.

(1 mark)

c) An employee increased the average temperature of the electronic thermostat by 3°C. Which of the following equations represents the new function?

Select the best answer.

(1 mark)

A) $y = \sin(0.26x - 2.88) + 19$

C) $y = 3 \sin(0.26x - 2.88) + 22$

B) $y = 3 \sin(0.26x - 2.88) + 16$

D) $y = 3 \sin(0.26x - 5.88) + 19$

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Given the equation $y = 400(0.9)^x$, describe what "400" could represent in a real-life situation.

Question 3   **Total: 3 marks**

Josemaría is standing directly under the centre of a parabolic satellite dish.

side view

- The bottom of the dish is 28 metres above the ground from where she is standing.
- The highest points of the dish are 50 m above the ground.
- The width of the dish is 64 m.

Diagram is not drawn to scale.
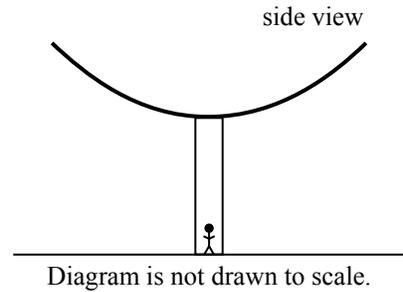
a) Determine a quadratic equation that models the shape of the dish. Show your work. You may use the table below.

(2 marks)

Horizontal Distance (m)	Height of the Dish (m)

b) Josemaría walks 20 metres to the right. She notices that a bird has made a nest on the dish directly above where she stands. Using your equation in (a), determine the height of the bird's nest from the ground.

(1 mark)

Question 4   Total: 6 marks
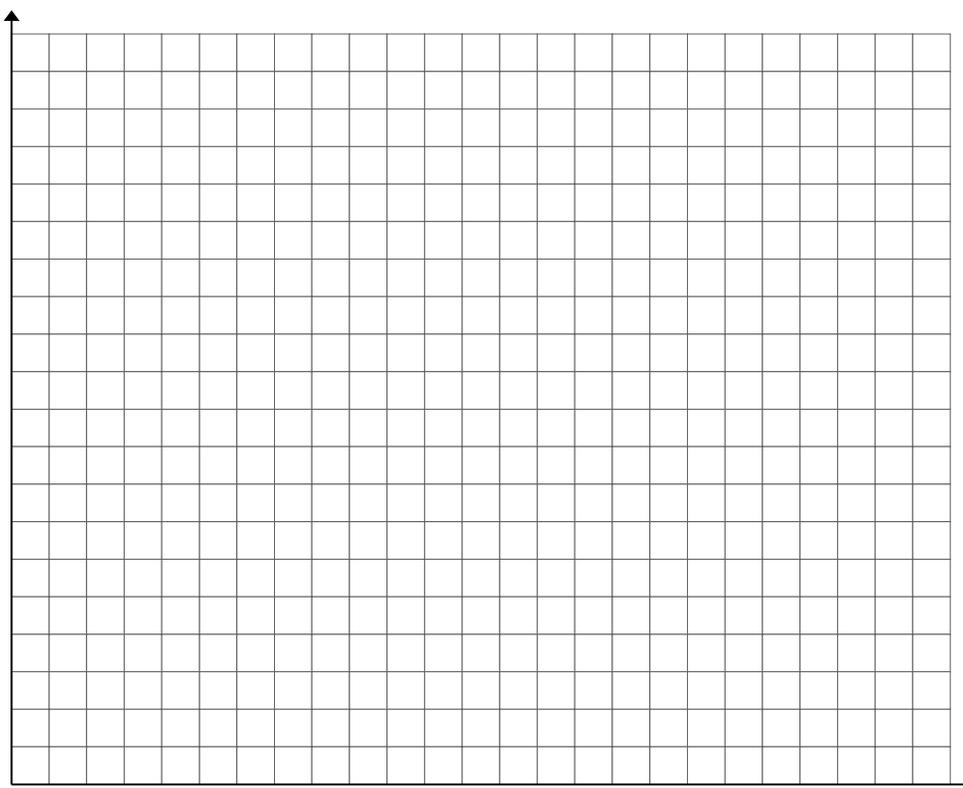
Corinne counts the number of large apples on her apple tree. The number of large apples increases over time as shown in the table below.

Time (days)	1	40	60	80	100	120
Number of large apples	0	66	76	82	87	92

a) Create a clearly labelled graph by plotting the given data. Draw a curve of best fit.

(3 marks)

b) Determine the logarithmic regression equation that models this data.

(1 mark)

c)   Corinne needs 84 large apples to make pies. How much time will it take for her to have enough apples? Show your work.

(2 marks)

A scientific experiment is conducted on a beverage containing aspartame.

- The initial amount of aspartame is 160 mg.
- The amount of aspartame decreases by 50% every 3 days.

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a) Determine an exponential equation that models the remaining amount of aspartame as a function of time (in days). Show your work.

(2 marks)

111

b) State the range for this experiment.

(1 mark)

PROBABILITY

Question 6 **Total: 1 mark**

Select the best answer.

A coin is flipped twice.

What is the probability of the coin landing on heads exactly two times?

A) 1

B) 0.75

C) 0.50

D) 0.25

113

The odds in favour of Danny's team winning the tournament are $3 : 7$.

Determine the probability that Danny's team does not win the tournament.

114

Using the digits 0 through 9, determine the number of 4-digit codes divisible by 5 that can be created if none of the digits repeat. Assume codes can start with zero. Show your work.

Martha is entering the Hudson Bay Quest sled dog race. She owns 12 dogs and wants to enter a team of 10 dogs.

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a)   Determine the number of ways she can randomly choose 10 of her dogs to make up a team.
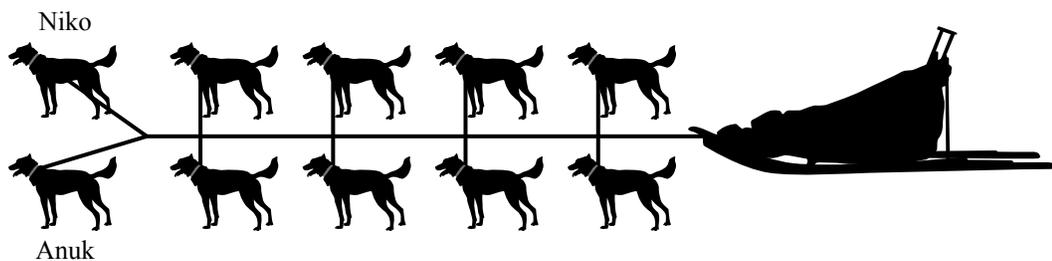
(1 mark)

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b)   Martha must now put the 10 chosen dogs in their starting positions. She begins by attaching her lead dogs, Niko and Anuk. Determine the number of ways she can randomly attach the remaining dogs.

(1 mark)

At recess, students randomly pick one marble from a bag to determine teams for a game. Initially, there are 10 orange marbles and 10 blue marbles in the bag.

Maria and Leah hope to be on the blue team. Maria picks her marble first and puts it in her pocket. Leah picks her marble second. What is the probability that they will both pick a blue marble? Show your work.

Question 11   Total: 3 marks

The weather report calls for a 60% probability of snow in northern Manitoba on Tuesday. The flight from Thompson to Flin Flon has a 30% probability of being on time when it is snowing. There is an 85% probability of the flight being on time when it is not snowing.

a)   Use a graphic organizer to show all possible outcomes for this situation.

(1 mark)

b)   Determine the probability that the flight on Tuesday will not be on time. Show your work.

(2 marks)

Question 12   Total: 4 marks

There are 5 jazz dancers and 7 ballet dancers from which 4 dancers are randomly chosen to form a group.

a) Determine the number of ways 4 dancers can be chosen.

(1 mark)

b) Determine the probability that 4 jazz dancers will be chosen. Show your work.

(2 marks)

c) Determine the probability that at least 1 ballet dancer will be chosen.

(1 mark)

Question 13   **Total: 1 mark**

Select the best answer.   123

The Mathletica company manufactures inflatable exercise balls. If the cost of plastic for the balls is $0.002/cm^2, which equation could be used to estimate the plastic cost, *C*, of one exercise ball with radius, *r*?

A) $C = 0.002 \times 4\pi r^2$
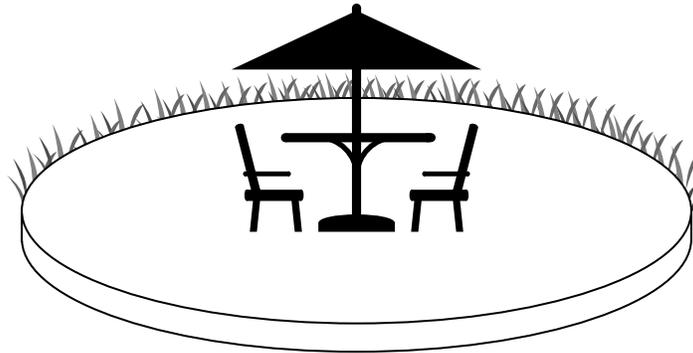
B) $C = 0.002 \times \dfrac{4}{3}\pi r^3$

C) $C = \dfrac{4\pi r^2}{0.002}$

D) $C = \dfrac{4\pi r^3}{3(0.002)}$

Question 14   Total: 3 marks

Kami would like to build a circular cement patio with a diameter of 15 feet as shown below. (Diagram is not drawn to scale.)

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a) Cement costs $200.00 per cubic yard, taxes included. Kami has a budget of $600.00 for the patio. Determine how many cubic feet of cement she can buy.

(1 mark)

125

b) What is the maximum height of the patio, in inches, that keeps her within budget? Show your work.
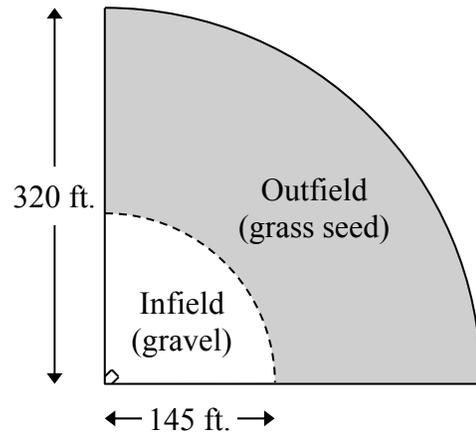
(2 marks)

Thierry wants to build a baseball field by his home. He is going to place gravel, to a depth of 4 inches, in the infield and grass seed on top of the existing soil in the outfield. (Diagram is not drawn to scale.)

The costs are as follows:

- $1.50 per cubic foot of gravel
- $16.00 per bag of grass seed, covering 6500 square feet

All items must be purchased in whole units and all prices are taxes included.

320 ft.

Outfield (grass seed)

Infield (gravel)

← 145 ft. →

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a) Determine the total cost to build the baseball field. Assume the field is in the shape of a quarter-circle.

(4 marks)

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b) Thierry obtains a loan from the bank to build the field described in (a). If he makes monthly payments of $400.00, how many months will it take to repay the loan at an interest rate of 6.25%, compounded monthly? Show your work.

(2 marks)

Question 16   Total: 2 marks

Samira's financial planner sent her an annual report of the investments in her portfolio. It stated that her $2000.00 stock investment increased in value by 6.00% while her $3000.00 mutual fund investment decreased in value by 8.00%.

Type of Investment	Principal ($)	Return ($)	Current Value ($)
Stocks	2000.00		
Mutual Funds	3000.00		
Total	5000.00		

Calculate the average rate of return of her portfolio. Show your work.

Question 17   Total: 2 marks

Your grandmother has some money that she would like to invest on your behalf. She would like to know whether you prefer a long-term or a short-term investment.

Explain one advantage for each type of investment.

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Salwa bought a new computer system for $6000.00. She anticipates the value of the system to depreciate at a rate of 15% per year.

What will the computer system be worth at the end of 3 years? Show your work.

Question 19   **Total: 4 marks**

Diego plans to move to Portage la Prairie. He has two housing options:

Option 1:

- Buy a house with a monthly mortgage payment of $1063.65.
- Property taxes are $3070.00 per year.

Option 2:

- Rent a house for $1250.00 per month.

a) What will be the total cost of Option 1 at the end of 25 years? Show your work.

(2 marks)

b) How many years will it take for the total cost of renting to equal the total cost of buying the house?

(1 mark)

c) Explain why Diego might choose to buy the house.

(1 mark)

Question 20   Total: 5 marks

Donald and Alex have a combined gross monthly income of $5500.00. They want to buy a house in a neighbourhood where the average monthly heating cost is $200.00 and monthly property taxes are $325.00.

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a) Calculate the maximum monthly mortgage payment they can afford, based on the gross debt service ratio. Show your work.

(2 marks)

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b) Based on the maximum monthly mortgage payment in (a), their bank has offered them a 25-year mortgage at an interest rate of 3.50%, compounded semi-annually. If they have saved $20 000.00 for a down payment, what would be the maximum house price they can afford? Show your work.
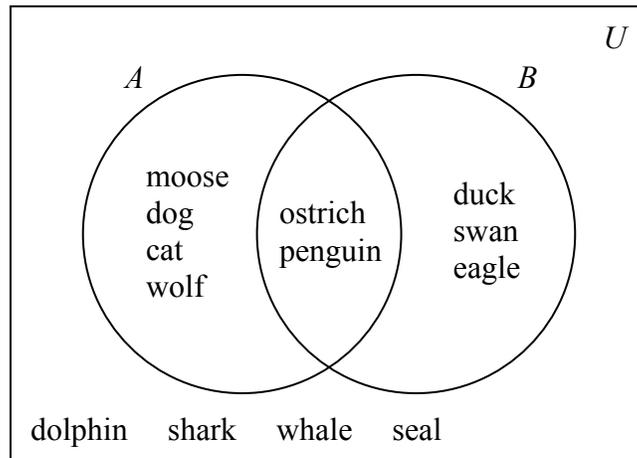
(3 marks)

LOGICAL REASONING

Question 21   **Total: 1 mark**

Use the following information to answer the question and select the best answer.

Which of the following statements is true?

A) $n(A \cup B) = 2$

B) $n(A' \cup B') = 4$

C) $n(A' \cap B') = 4$

D) $n(A \cap B) = 9$

You are given the following four inequality statements.

I. $\bigstar < \triangle$

II. $\bigcirc \neq \bigstar$

III. $\bigstar > \square$

IV. $\bigcirc < \square$

Draw the four symbols in order from smallest value to largest value.

Question 23   Total: 4 marks

A technology survey of 50 students found that:

- 30 students use Instagram
- 25 students use Snapchat
- 15 students use Facebook
- 8 students use only Instagram and Snapchat
- 3 students use only Facebook and Instagram
- 4 students use only Facebook and Snapchat
- 6 students use all three apps

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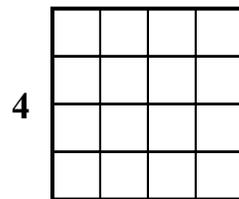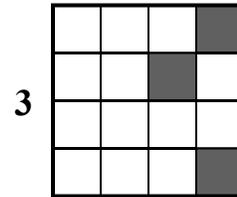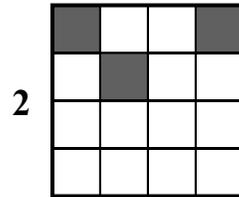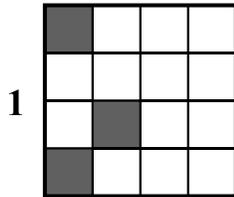a)   Draw a Venn diagram to represent this situation.

(3 marks)

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b)   How many students use Snapchat or Instagram?

(1 mark)

Question 24   Total: 1 mark

Use the following pattern to draw the 4th picture.

END OF TEST

**NO MARKS WILL BE AWARDED
FOR WORK DONE ON THIS PAGE.**

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FOR WORK DONE ON THIS PAGE.**

Formula Sheet: Applied Mathematics

Relations and Functions	Financial Mathematics
$y = ax + b$	$t = \dfrac{72}{i}$
$y = ax^2 + bx + c$	$I = Prt$
$y = ax^3 + bx^2 + cx + d$	$A = P\left(1 + \dfrac{r}{n}\right)^{nt}$
$y = ab^x$	Net worth = Total assets − Total liabilities
$y = a + b\ln(x)$	Debt-to-equity ratio (%) $= \dfrac{(\text{Total liabilities} - \text{Mortgage})}{\text{Net worth}} \times 100$
$y = a\log_b x$	
$y = a\sin(bx + c) + d$	Gross debt service ratio (%) $= \dfrac{\left(\begin{array}{ccc}\text{Monthly} & \text{Monthly} & \text{Monthly}\\ \text{mortgage} + \text{property} + \text{heating}\\ \text{payment} & \text{taxes} & \text{costs}\end{array}\right)}{\text{Gross monthly income}} \times 100$
$y = a\cos(bx + c) + d$	
	Rate of return (%) $= \dfrac{\left(\begin{array}{cc}\text{Current value} & \text{Previous value}\\ \text{of portfolio} & \text{of portfolio}\end{array}\right)}{\text{Previous value of portfolio}} \times 100$

Probability	Design and Measurement
$P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$	Prism: Surface area $= Ph + 2B$
	Volume $= Bh$
$P(A \text{ and } B) = P(A) \times P(B)$	Pyramid: Surface area $= B + \dfrac{Ps}{2}$ (s = slant height)
$P(A \text{ and } B) = P(A) \times P(B|A)$	Volume $= \dfrac{Bh}{3}$
$_nP_r = \dfrac{n!}{(n-r)!}$	Sphere: Surface area $= 4\pi r^2$
	Volume $= \dfrac{4}{3}\pi r^3$
$_nC_r = \dfrac{n!}{r!(n-r)!}$	Cylinder: Surface area $= 2\pi rh + 2\pi r^2$
	Volume $= \pi r^2 h$
	Cone: Surface area $= \pi r^2 + \pi rs$
	Volume $= \dfrac{\pi r^2 h}{3}$