Chapter 5 Graphing inequalities & Systems of equations

❖ *A review is a brief summary of the chapter. It is never an acceptable substitute for learning and understanding the notes, homework and assignments throughout the entire chapter.*

e.g. 1

Determine which of the ordered pairs are solutions to the given inequality.

$3x - 2y > 12$ $\{(6, 3), (12, -4), (-6, -1)\}$ ✱ test each point

$(6, 3)$

$3(6) - 2(3) > 12$

$12 > 12$

False

(Fail)

$(12, -4)$

$3(12) - 2(-4) > 12$

$36 + 8 > 12$

$44 > 12$ ✓

True

(Pass)

$(-6, -1)$

$3(-6) - 2(-1) > 12$

$-18 + 2 > 12$

$-16 > 12$

False

(Fail)

$A(12, -4)$ is the only solution

e.g. 2

Graph the following inequalities.

a) $3x + y > -2$

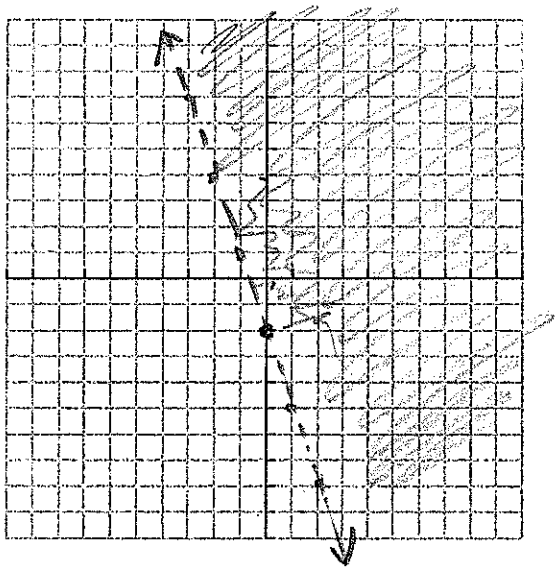
Solve for y

$y > -3x - 2$

Test $(0, 0)$

$3(0) + 0 > -2$

$0 > -2$ ✓ True. Verified.



b) $4x - 3y + 6 \leq 0$

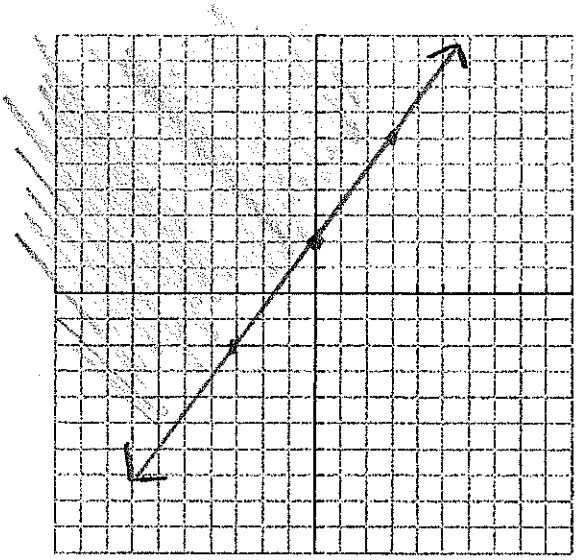
$-3y \leq -4x - 6$

$y \geq \frac{4}{3}x + 2$

Test $(0, 0)$ Should fail

$4(0) - 3(0) + 6 \leq 0$

$6 \leq 0$ False ✓

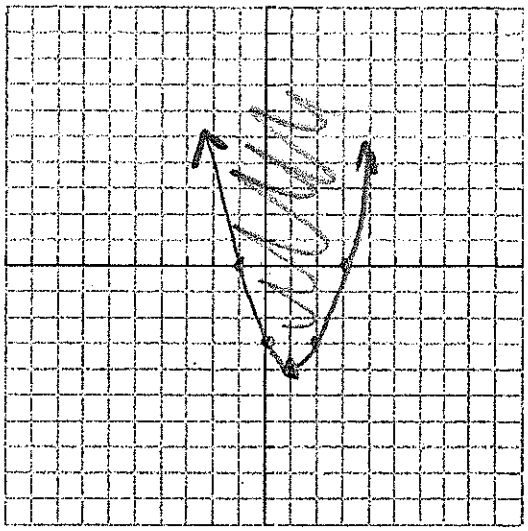


c) $y \geq x^2 - 2x - 3$

$y \geq (x-1)^2 - 4$

$y \geq (x-3)(x+1)$

x-int $x = 3, -1$



Test $(0,0)$ Shaded pass

$0 \geq (0)^2 - 2(0) - 3$

$0 \geq -3$ ✓

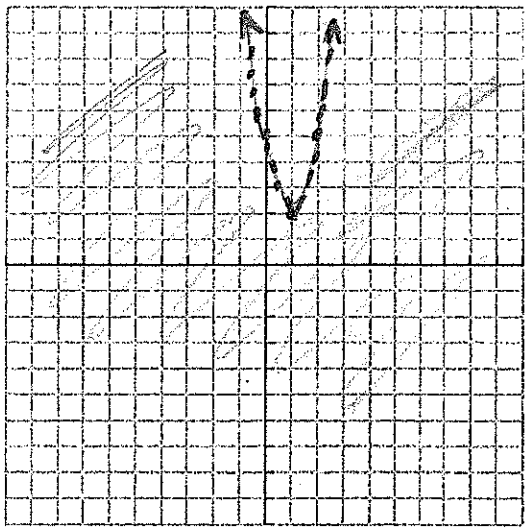
d) $y < 3x^2 - 6x + 5$

$y < 3(x^2 - 2x \quad) + 5$

$y + 3 < 3(x^2 - 2x + 1) + 5$

$y < 3(x-1)^2 + 2$

$V: (1, 2)$



Test $(0,0)$ (pass)

$0 < 3(0)^2 - 6(0) + 5$

$0 < 5$ ✓

e.g. 3

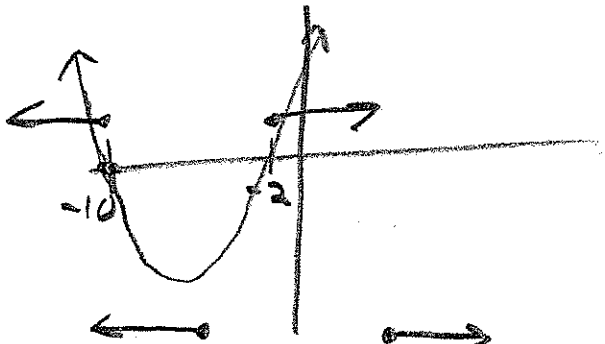
Solve the following quadratic inequalities. (Draw a rough sketch—it will help)

a) $x^2 + 12x + 20 \geq 0$

* Factor

$(x+10)(x+2) \geq 0$

x-int $x = -10, -2$



Test points to Verify

-11	-3	0
$(11)^2 + 12(11) + 20 \geq 0$	$(-3)^2 + 12(-3) + 20 \geq 0$	$(0)^2 + 12(0) + 20 \geq 0$
$9 \geq 0$ ✓	$-7 \geq 0$	$20 \geq 0$ ✓
pass	fail ✗	pass.

Soln

$x \leq -10$ or $x \geq -2$

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b) $3x^2 + 7x < -2$

move everything to one side & factor

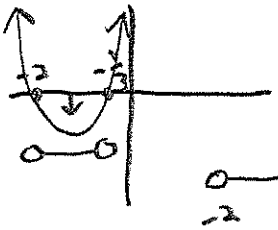
$3x^2 + 7x + 2 < 0$

$3x^2 + 6x + x + 2 < 0$

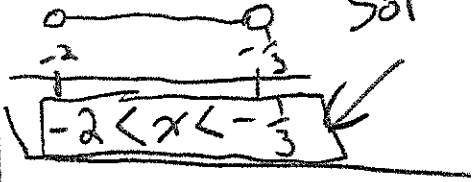
$3x(x+2) + (x+2) < 0$

$(3x+1)(x+2) < 0$

x-int $x = -\frac{1}{3}, x = -2$



Soln



Verify

$x = -3$	$x = -1$	$x = 0$
$3(-3)^2 + 7(-3) < -2$	$3(-1)^2 + 7(-1) < -2$	$3(0)^2 + 7(0) < -2$
$6 < -2$	$-4 < -2$	
✗	✓	

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c) $2x^2 > 9x - 9$

* move everything to one side & factor

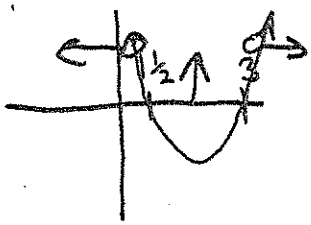
$2x^2 - 9x + 9 > 0$

$2x^2 - 6x - 3x + 9 > 0$

$2x(x-3) - 3(x-3) > 0$

$(2x-3)(x-3) > 0$

x-int $x = \frac{3}{2}$ or $1\frac{1}{2}, x = 3$



Soln $x < 1\frac{1}{2}, x > 3$

Verify

$x = 0$	$x = 2$	$x = 4$
$2(0)^2 > 9(0) - 9$	$2(2)^2 > 9(2) - 9$	$2(4)^2 > 9(4) - 9$
$0 > -9$	$8 > 9$	$32 > 27$
✓	✗	✓

e.g. 4

Create an inequality that has the solution $x < 4$ and $x > -2$. Show solution in general form.

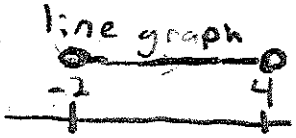
An equation with x-int $x=4$ & $x=-2$ is

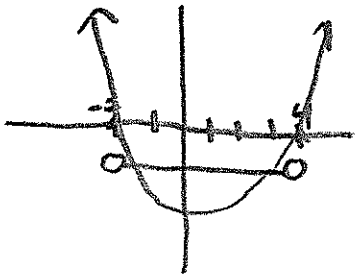
$(x-4)(x+2)=0$

$\Downarrow$

$(x-4)(x+2)<0$

make line graph





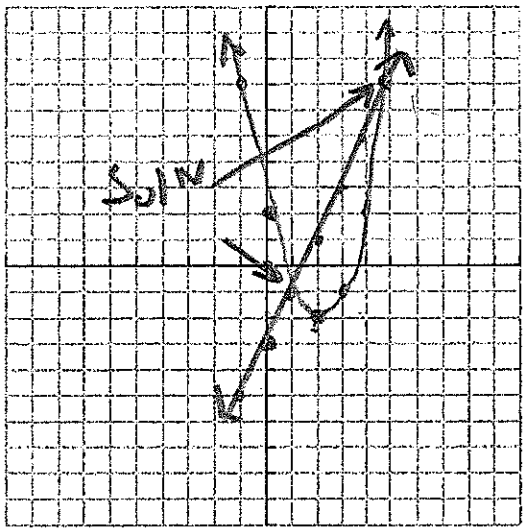
e.g. 5

Solve the following systems graphically and algebraically

a) $y = (x-2)^2 - 2$
$y = 2x - 3$

graph each.

V: (2, -2)



Soln (1, -1) or (5, 7)

use Substitution

$y=(x-2)^2-2$

$y=(2x-3)$

Foil $(x-2)(x-2)$

$2x-3=(x-2)^2-2$

$2x-3=x^2-4x+4-2$

$2x-3=x^2-4x+2$

$0=x^2-6x+5$

$0=(x-5)(x-1)$

$x=5$ or $x=1$

Sub $x=5$ & $x=1$ into original

$x=5$

$y=2x-3$

$y=2(5)-3$

$y=7$

$x=1$

$y=2(1)-3$

$y=-1$

Soln (5, 7) or (1, -1)

E.g. 6.

Two numbers are related below:

When twice the second number is subtracted from the first number, the result is -11

The square of the sum of the first number and 2 is equal four less than the second number

a) Create a system of equations to represent this inequality (2 marks)

Let $x = 1^{st}$ #

$y = 2^{nd}$ #

① $x - 2y = -11$

② $(x+2)^2 = y - 4$

b) Solve the system to determine the numbers (2 marks)

- Solve for y in ②

$y = (x+2)^2 + 4$

- Sub into equation ①

$x - 2((x+2)^2 + 4) = -11$

- Solve for x

$x - 2(x^2 + 4x + 4 + 4) = -11$

$x - 2(x^2 + 4x + 8) = -11$

$x - 2x^2 - 8x - 16 = -11$

$-2x^2 - 7x - 5 = 0$

$-(2x^2 + 7x + 5) = 0$

$-(2x^2 + 5x + 2x + 5) = 0$

$-(x(2x+5) + (2x+5)) = 0$

$-(x+1)(2x+5) = 0$

$x = -1$ or $x = -\frac{5}{2}$ or $-2\frac{1}{2}$

Sub into an original to get y's

$x = -1$: $(-1) - 2y = -11$, $-2y = -10$, $y = 5$

$x = -\frac{5}{2}$: $(-\frac{5}{2}) - 2y = -11$, $-2y = -8\frac{1}{2}$, $y = 4\frac{1}{4}$

SOLN $(-1, 5)$ or $(-2\frac{1}{2}, 4\frac{1}{2})$