Chapter 3 Review   Name ___Key_____

1. Explain the difference between what is being asked in the following questions and complete the questions.

a) Factor

$$x^2 + 5x + 6$$

$$(x+2)(x+3)$$

Just factor. - can't find x

b) Solve by Factoring

$$x^2 + 7x + 6 = 0$$

$$(x+6)(x+1) = 0$$

$$x+6=0 \text{ or } x+1=0$$

$$x=-6 \qquad x=-1$$

"solve" (& given that there is an "=" sign, you can find x.

2. What is an extraneous root? Give an example.

3. What is the discriminant? What does the discriminant tell you? Give examples.

Discriminant - the radicand in the quadratic formula.

If $b^2-4ac = 0$
↓
1 solution

If $b^2-4ac > 0$
2 solutions

If $b^2-4ac < 0$
0 solutions

4. Factor the following polynomials completely.

(a) $9(x-1)^2 - 100y^2$

$$= (3(x-1))^2 - (10y)^2$$

$$= (3(x-1)+10y)(3(x-1)-10y)$$

$$= (3x-3+10y)(3x-3-10y)$$

(b) $\frac{1}{4}x^2 + \frac{1}{2}x - 6$

$$= \frac{1}{4}x^2 + \frac{2}{4}x - \frac{24}{4}$$

$$= \frac{1}{4}(x^2+2x-24)$$

$$= \frac{1}{4}(x+6)(x-4)$$

(c) $0.1n^2 - 0.1n - 3$

$$= 0.1(n^2 - n - 30)$$

$$= 0.1(n-6)(n+5)$$

(d) $4(x+3)^2 + 8(x+3) - 5$

$$4a^2 + 8a - 5$$

$$= 4a^2 + 10a - 2a - 5$$

$$= 2a(2a+5) - 1(2a+5)$$

$$= (2a+5)(2a-1)$$

$$= (2(x+3)+5)(2(x+3)-1)$$

$$= (2x+6+5)(2x+6-1) = (2x+11)(2x+5)$$

$a = x+3$

$mn = 4(-5) = -20$

10 -2

5. Solve each equation by factoring.

(a) $x^2 + 7x + 10 = 0$

$(x+5)(x+2)=0$

$x+5=0$ or $x+2=0$
  $-5\ -5$ $-2\ -2$

$x=-5$ $x=-2$

(b) $x^2 - x = 6$

$x^2 - x - 6 = 0$

$(x-3)(x+2)=0$

$x-3=0$ $x+2=0$

$x=3$ $x=-2$

(c) $8x^2 = 72x - 144$

$$\frac{8x^2 - 72x + 144 = 0}{8} \quad \frac{}{8}$$

$x^2 - 9x + 18 = 0$

$(x-6)(x-3)=0$

$x=6$ $x=3$

(d) $5x^2 + 20 = -25x$

$$\frac{5x^2 + 25x + 20 = 0}{5} \quad \frac{}{5}$$

$x^2 + 5x + 4 = 0$

$(x+4)(x+1)=0$

$x=-4$ $x=-1$

$mn = 4(3)$
$\quad = 12$

$\dfrac{6x^2 = 12}{6} \quad \dfrac{}{6+2} = 8$

(e) $4x^2 + 8x + 3 = 0$

$4x^2 + 6x + 2x + 3 = 0$

$2x(2x+3) + 1(2x+3) = 0$

$(2x+3)(2x+1)=0$

$2x+3=0$ $2x+1=0$

$x=\dfrac{-3}{2}$ $x=-\dfrac{1}{2}$

(f) $2x^2 - 5x = 0$

$x(2x-5)=0$

$x=0$ $2x-5=0$
       $2x=5$

       $x=\dfrac{5}{2}$

6. Write a quadratic equation that has the following solutions.

(a) -5, 7

$x=-5$ $x=7$

$x+5=0$ $x-7=0$

$(x+5)(x-7)=0$

$x^2 - 7x + 5x - 35 = 0$

$x^2 - 2x - 35 = 0$

(b) $2, \dfrac{4}{3}$ $x = \dfrac{4}{3}$

$x=2$ $3x=4$

$x-2=0$ $3x-4=0$

$(x-2)(3x-4)=0$

$3x^2 - 4x - 6x + 8 = 0$

$3x^2 - 10x + 8 = 0$

7. Solve each equation.

(a) $8x^2 - 7 = 249$

$$\frac{+7 \quad +7}{8x^2 = 256}$$
$$\frac{8x^2}{8} = \frac{256}{8}$$
$$x^2 = 32$$

$x = \pm\sqrt{32}$
$x = \pm\sqrt{16 \cdot 2}$
$\boxed{x = \pm 4\sqrt{2}}$

(b) $(x+5)^2 = 49$

$x + 5 = \pm\sqrt{49}$
$x = -5 \pm 7$

$x = -5 + 7 = \boxed{2}$
$x = -5 - 7 = \boxed{-12}$

(c) $\dfrac{2(x-2)^2}{2} = \dfrac{18}{2}$

$(x-2)^2 = 9$
$x - 2 = \pm\sqrt{9}$
$x = 2 \pm 3$

$x = 2 + 3$
$\boxed{5}$
$x = 2 - 1$
$\boxed{= -1}$

(d) $\left(x - \dfrac{7}{5}\right)^2 = \dfrac{36}{25}$

$x - \dfrac{7}{5} = \pm\sqrt{\dfrac{36}{25}}$
$x = \dfrac{7}{5} \pm \dfrac{6}{5}$

$x = \dfrac{7}{5} + \dfrac{6}{5} = \boxed{\dfrac{13}{5}}$
$x = \dfrac{7}{5} - \dfrac{6}{5} = \boxed{\dfrac{1}{5}}$

8. Solve each equation by completing the square.

(a) $\dfrac{3x^2 - 12x + 9}{3} = \dfrac{0}{3}$

$\frac{1}{2}(-4) = -2$
$\rightarrow (-2)^2 = 4$

$x^2 - 4x + 3 = 0$
$x^2 - 4x + 4 - 4 + 3 = 0$
$(x^2 - 4x + 4) - 1 = 0$
$(x-2)^2 = 1$

$x - 2 = \pm\sqrt{1}$
$x = 2 + 1$
$\boxed{x = 3}$
$x = 2 - 1$
$\boxed{x = 1}$

(b) $x^2 - 12x + 31 = 0$

$\frac{1}{2}(-12) = -6$
$\rightarrow (-6)^2 = 36$

$x^2 - 12x + 36 - 36 + 31 = 0$
$(x^2 - 12x + 36) - 5 = 0$
$(x-6)^2 = 5$
$x - 6 = \pm\sqrt{5}$
$\rightarrow x = 6 \pm \sqrt{5}$

(c) $\dfrac{-4x^2 + 24x - 21}{-4} = \dfrac{0}{-4}$

$\frac{1}{2}(-6) = -3$
$\rightarrow (-3)^2 = 9$

$x^2 - 6x + \dfrac{21}{4} = 0$
$x^2 - 6x + 9 - 9 + \dfrac{21}{4} = 0$
$(x^2 - 6x + 9) - \dfrac{36}{4} + \dfrac{21}{4} = 0$

$(x-3)^2 = \dfrac{15}{4}$
$x - 3 = \pm\sqrt{\dfrac{15}{4}}$
$x = 3 \pm \dfrac{\sqrt{15}}{2}$

(d) $\left(\dfrac{1}{4}x^2 + x - \dfrac{7}{2}\right) = 0 \times 4$

$\frac{1}{2}(4) = 2$
$\rightarrow (2)^2 = 4$

$x^2 + 4x - 14 = 0$
$x^2 + 4x + 4 - 4 - 14 = 0$
$(x^2 + 4x + 4) - 20 = 0$
$(x+2)^2 = 20$

$x + 2 = \pm\sqrt{20}$
$x = -2 \pm \sqrt{20}$
$= -2 \pm 2\sqrt{5}$

9. Solve each equation with the quadratic formula.

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

(a) $4x^2 - 3x - 27 = 0$

$a = 4 \quad b = -3 \quad c = -27$

$x = \dfrac{-(-3) \pm \sqrt{(-3)^2 - 4(4)(-27)}}{2(4)}$

$= \dfrac{3 \pm \sqrt{9 + 432}}{8}$

$= \dfrac{3 \pm \sqrt{441}}{8}$

$x = \dfrac{3 + 21}{8}$
$\boxed{= 3}$
$x = \dfrac{3 - 21}{8}$
$\boxed{= \dfrac{-9}{4}}$

(b) $x^2 - 10x + 22 = 0$

$a = 1 \quad b = -10 \quad c = 22$

$x = \dfrac{-(-10) \pm \sqrt{(-10)^2 - 4(1)(22)}}{2(1)}$

$= \dfrac{10 \pm \sqrt{100 - 88}}{2}$

$= \dfrac{10 \pm \sqrt{12}}{2}$

$= \dfrac{10 \pm 2\sqrt{3}}{2}$

$\rightarrow x = 5 \pm \sqrt{3}$

10. Use the discriminant to determine the number of solutions to each question. $b^2 - 4ac$

(a) $2x^2 - 9x + 4 = 0$

$(-9)^2 - 4(2)(4)$
$= 81 - 32$
$= 49$

2 solutions

(b) $-6x^2 + 7x - 5 = 0$

$(7)^2 - 4(-6)(-5)$
$= 49 - 120$
$=$

no solutions

(c) $-6x^2 - 3x + 9 = 0$

$(-3)^2 - 4(-6)(9)$
$= 9 + 216$
$= 225$

2 solutions

(d) $-x^2 - 6x - 9 = 0$

$= (-6)^2 - 4(-1)(-9)$
$= 36 - 36$
$= 0$

1 solution

11. Solve the following.

(a) $x - 1 = \dfrac{2}{x}$

$x(x-1) = x\left(\dfrac{2}{x}\right)$

$x^2 - x = 2$

$x^2 - x - 2 = 0$

$(x-2)(x+1) = 0$

$x = 2 \quad x = -1$

(b) $x(2x - 3) + 4(x + 1) = 2(3 + 2x)$

$2x^2 - 3x + 4x + 4 = 6 + 4x$

$2x^2 + x + 4 = 6 + 4x$
$\quad -4x \quad -6 \quad -6 \quad -4x$

$2x^2 - 3x - 2 = 0$

$2x^2 - 4x + x - 2 = 0$

$2x(x-2) + 1(x-2) = 0$

$(x-2)(2x+1) = 0$

$mn = 2(-2) = -4$
$\quad \overset{\wedge}{}$
$\quad -4 \ +1$

$x - 2 = 0$
$x = 2$

$2x + 1 = 0$
$x = -\dfrac{1}{2}$

(c) $-2 - 3(x + 1)^2 = -50$
$\ +2 \qquad\qquad\qquad +2$

$\dfrac{-3(x+1)^2}{-3} = \dfrac{-48}{-3}$

$(x+1)^2 = 16$

$x + 1 = \pm\sqrt{16}$

$x = -1 \pm 4$

$x = -1 + 4 \qquad x = -1 - 4$

$x = 3 \qquad\qquad x = -5$

(d) $2(x + 3)^2 - 11(x + 3) + 15 = 0$

$a = x + 3$

$mn = 2(15)$
$= 30$
$\quad \overset{\wedge}{}$
$\quad -5 \ -6$

$2a^2 - 11a + 15 = 0$

$2a^2 - 5a - 6a + 15 = 0$

$a(2a-5) - 3(2a-5) = 0$

$(2a-5)(a-3) = 0$

$(2(x+3) - 5)(x+3-3) = 0$

$(2x+6-5)(x) = 0$

$(2x+1)(x) = 0$

$x = -\dfrac{1}{2} \qquad x = 0$

(e) $\frac{1}{4}x^2 + \frac{1}{2}x = 1$

$4\left(\frac{1}{4}x^2 + \frac{1}{2}x - 1\right) = 0 \times 4$

$x^2 + 2x - 4 = 0$

$x = \frac{-2 \pm \sqrt{2^2 - 4(1)(-4)}}{2(1)}$

$= \frac{-2 \pm \sqrt{20}}{2}$ $\Bigg\} x = -1 \pm \sqrt{5}$

$= \frac{-2 \pm 2\sqrt{5}}{2}$

(f) $\sqrt{2x - 7} + 5 = x$

$ -5 -5$

$\sqrt{2x-7}^2 = (x-5)^2$
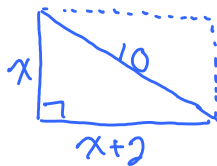
$2x - 7 = (x-5)(x-5)$

$2x - 7 = x^2 - 10x + 25$

$-2x + 7 -2x + 7$

$0 = x^2 - 12x + 32$

$0 = (x-4)(x-8)$

$x = 4 \quad x = 8$

11. The diagonal of a rectangle is 10 cm. The length is 2 cm longer than the width. Determine the width.

$x^2 + (x+2)^2 = 10^2$

$x^2 + (x+2)(x+2) = 100$

$x^2 + x^2 + 4x + 4 = 100$

$\frac{2x^2 + 4x - 96 = 0}{2} \frac{}{2}$

$x^2 + 2x - 48 = 0$

$(x+8)(x-6) = 0$

$\cancel{x = -8} \leftarrow$ width can't be negative

$x = 6$

width = 6 cm

length = 8 cm

11. The height of a golf ball, in yards, is $h(d) = -0.02d^2 + 2d$, where d is the horizontal distance the ball has travelled, in yards, after being struck. Determine how far the ball travels before it first strikes the ground. (Note: Think of what the height is when it touches the ground).

$\leftarrow$ height when it touches the ground is 0 yards.

$h = -0.02d^2 + 2d$

$0 = -0.02d^2 + 2d$

$0 = -0.02d(d - 100)$

$d = 0 \qquad d = 100$

The ball travels 100 m.