Chapter 4 Analyzing Quadratic Functions

- *A review is a brief summary of the chapter. It is never an acceptable substitute for learning and understanding the notes, homework and assignments throughout the entire chapter.*

Know how to complete the square to change to vertex(standard) form, factor to change to factored form, and be able to pick characteristics from the different forms of quadratics.

e.g. 1

Determine the following characteristics for each function.

- The vertex
- The domain and range
- The direction of opening
- The equation of the axis of symmetry

Then sketch the graph

a) $y = 2(x+1)^2 - 3$

Vertex: $(-1, -3)$

direction of opening: up

equation of axis of symmetry: $x = -1$

Domain: $x \in R$

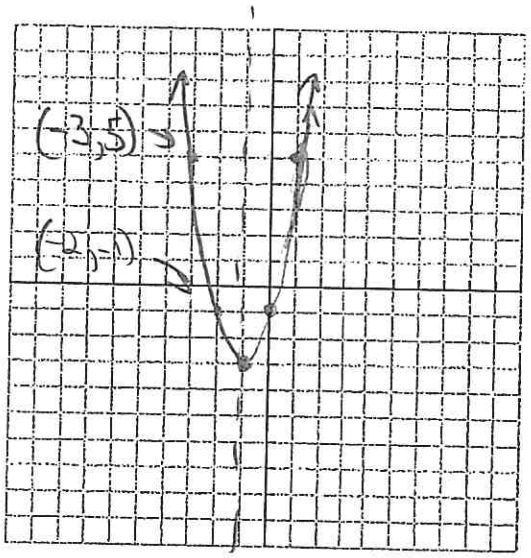
Range: $y \geq -3$

lets transform to find a couple more points

$(1,1) \xrightarrow{x2} (\bar{1}, \bar{2}) \xrightarrow{-3} (0, -1)$

$(2,4) \xrightarrow{x2} (\bar{2}, \bar{8}) \xrightarrow{-3} (1, 5)$

must start with points on $y = x^2$



b) $y = -\frac{1}{4}(x-4)^2 + 1$

V: $(4, 1)$

opens: down

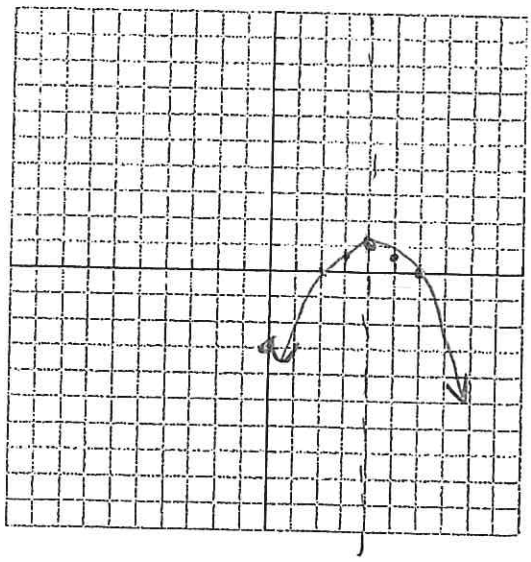
axis of symmetry: $x = 4$

Domain: $x \in R$

Range: $y \leq 1$

$(1,1) \xrightarrow{x-\frac{1}{4}} (1, -\frac{1}{4}) \xrightarrow{+4,+1} (5, \frac{3}{4})$

$(2,4) \xrightarrow{x-\frac{1}{4}} (2, -1) \xrightarrow{+4,+1} (6, 0)$



x-int are 2, 6

y-int: $y = -\frac{1}{4}(0-4)^2 + 1$

$y = -\frac{1}{4}(16) + 1 = -3$

e.g. 2

Determine a Quadratic Function for each of the following given the following information.

a) A parabola that has a vertex V(5, -4) and passes through the points (2 , -1) and (8 , -1)

Start with vertex form

$y = a(x-p)^2 + q$

$y = a(x-(5))^2 + 4$

$y = a(x-5)^2 - 4$

$-1 = a(2-5)^2 - 4$

$-1 = a(-3)^2 - 4$

$-1 = a(9) - 4$

$3 = a(9)$

$a = \frac{1}{3}$

$y = \frac{1}{3}(x-5)^2 - 4$

b) A parabola that has x- intercepts of -2 and 4 and passes through the point E(2, -16)

Start with factored form

$y = a(x-x_1)(x-x_2)$

$y = a(x-(-2))(x-(4))$

$y = a(x+2)(x-4)$

$-16 = a(2+2)(2-4)$

$-16 = a(4)(-2)$

$-16 = a(-8)$

$2 = a$

$y = 2(x+2)(x-4)$

e.g. 3

How many x – intercepts does each of these quadratic functions have

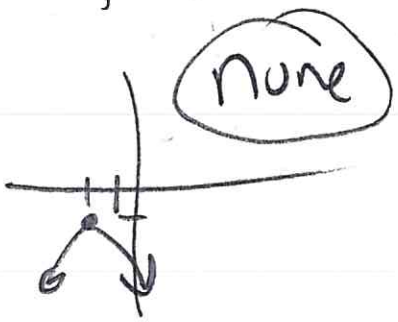
a) $f(x) = 2(x-1)^2$ +0

V(1, 0) opens up

1 x-intercept.

b) $f(x) = -3(x+2)^2 - 1$

V(-2, -1)



e.g. 4 Sketch each of the following and state the characteristics

a) $y = x^2 - 2x$

Can you factor?

$y = x(x-2)$

x-int

$0 = x(x-2)$

$x = 0 \quad x = 2$

Vertex

$y = (1)^2 - 2(1)$

$y = 1 - 2$

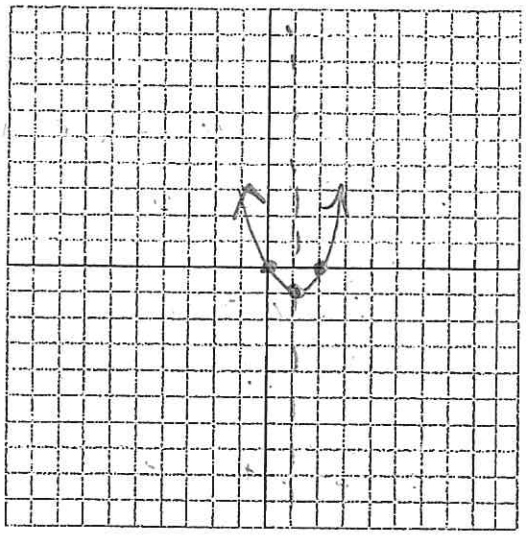
$y = -1$

$V(1, -1)$

Find axis of symmetry the use to find vertex

$\frac{x_1 + x_2}{2} = 1$

$x = 1$



b) $y = -x^2 + 2x + 8$

$y = -(x^2 - 2x - 8)$

$y = -(x-4)(x+2)$

$0 = -(x-4)(x+2)$

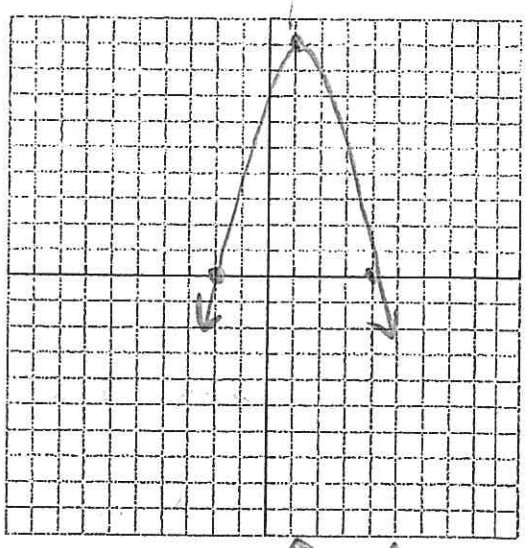
$x = 4 \quad x = -2$

axis of symmetry: $x = 1$

Vertex $y = -(1)^2 + 2(1) + 8$

$y = -1 + 2 + 8$

$y = 9 \quad V(1, 9)$



c) $y = 2x^2 - 12x + 25$

change to vertex form

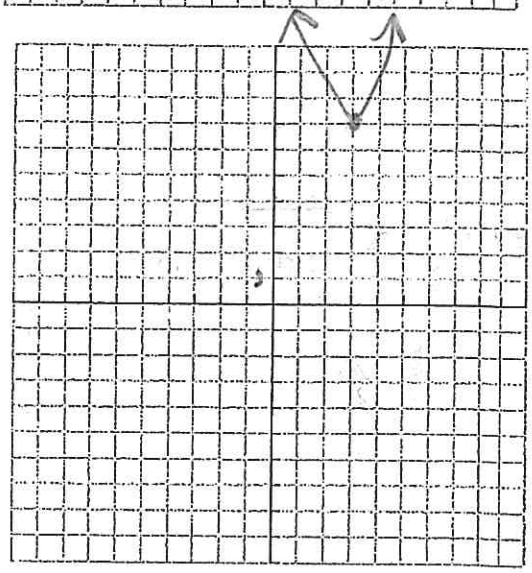
$y = 2(x^2 - 6x\ ) + 25$

$y + 18 = 2(x^2 - 6x + 9) + 25$

$y + 18 = 2(x-3)^2 + 25$

$y = 2(x-3)^2 + 7$

$V(3, 7)$



y-int is 25

e.g. 5

State the vertex ot $y = 5x^2 + 30x + 41$

Change to Vertex form

$y = 5(x^2 + 4x\ \ ) + 41$

$y \overset{+20}{=} 5(x^2 + 4x + 4) + 41$

$y = 5(x+2)^2 + 41 - 20$

$y = 5(x+2)^2 + 21$

e.g. 6

The manager of a clothing company is analysing its costs, revenues, and profits to plan for the upcoming year. Last year, a certain type of children's winter coat was priced at $40, and the company sold 10 000 of them. Market research says that for every $2 decrease in the price, the manager can expect the company to sell 500 more coats. Determine the maximum revenue and the price that will achieve that revenue.

Let x = # of decreases in price

Revenue = # items sold x price

$R = (10\,000 + 500x)(40 - 2x)$

$= 400\,000 - 20\,000x + 20\,000x - 1000x^2$

$= -1000x^2 + 400\,000$

$x=0$	$10\,000$	$40
$x=1$	$10\,000 + 500x$	$40 - 2x$

Tricky, This is Vertex form b/c no x term

$x=0$ ∴ no decreases will give the max Revenue.

A price of $40 will give a max revenue of $400 000