Grade 12
Pre-Calculus Mathematics
Achievement Test

Marking Guide

June 2014

Manitoba

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After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Text Book Bureau. Order online at <www.mtbb.mb.ca>.

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Websites are subject to change without notice.

Disponible en français.

Available in alternate formats upon request.

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General Marking Instructions

Please do not make any marks in the student test booklets. If the booklets have marks in them, the marks will need to be removed by departmental staff prior to sample marking should the booklet be selected.

Please ensure that

- the booklet number and the number on the *Answer/Scoring Sheet* are identical
- **students and markers use only a pencil to complete the *Answer/Scoring Sheets***
- the totals of each of the four parts are written at the bottom
- each student's final result is recorded, by booklet number, on the corresponding *Answer/Scoring Sheet*
- the *Answer/Scoring Sheet* is complete
- a photocopy has been made for school records

Once marking is completed, please forward the *Answer/Scoring Sheets* to Manitoba Education and Advanced Learning in the envelope provided (for more information see the administration manual).

Marking the Test Questions

The test is composed of short-answer questions, long-answer questions, and multiple-choice questions. Short-answer questions are worth 1 or 2 marks each, long-answer questions are worth 3 to 5 marks each, and multiple-choice questions are worth 1 mark each. An answer key for the multiple-choice questions can be found at the beginning of the section "Booklet 2 Questions."

To receive full marks, a student's response must be complete and correct. Where alternative answering methods are possible, the *Marking Guide* attempts to address the most common solutions. For general guidelines regarding the scoring of students' responses, see Appendix A.

Irregularities in Provincial Tests

During the administration of provincial tests, supervising teachers may encounter irregularities. Markers may also encounter irregularities during local marking sessions. Appendix B provides examples of such irregularities as well as procedures to follow to report irregularities.

If an *Answer/Scoring Sheet* is marked with "0" and/or "NR" only (e.g., student was present but did not attempt any questions), please document this on the *Irregular Test Booklet Report*.

Assistance

If, during marking, any marking issue arises that cannot be resolved locally, please call Manitoba Education and Advanced Learning at the earliest opportunity to advise us of the situation and seek assistance if necessary.

You must contact the Assessment Consultant responsible for this project before making any modifications to the answer keys or scoring rubrics.

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Communication Errors

The marks allocated to questions are primarily based on the concepts and procedures associated with the learning outcomes in the curriculum. For each question, shade in the circle on the *Answer/Scoring Sheet* that represents the marks given based on the concepts and procedures. A total of these marks will provide the preliminary mark.

Errors that are not related to concepts or procedures are called “Communication Errors” (see Appendix A) and will be tracked on the *Answer/Scoring Sheet* in a separate section. There is a ½ mark deduction for each type of communication error committed, regardless of the number of errors per type (i.e., committing a second error for any type will not further affect a student’s mark), with a maximum deduction of 5 marks from the total test mark.

The total mark deduction for communication errors for any student response is not to exceed the marks given for that response. When multiple communication errors are made in a given response, any deductions are to be indicated in the order in which the errors occur in the response, without exceeding the given marks.

The student’s final mark is determined by subtracting the communication errors from the preliminary mark.

Example: A student has a preliminary mark of 72. The student committed two E1 errors (½ mark deduction), four E7 errors (½ mark deduction), and one E8 error (½ mark deduction). Although seven communication errors were committed in total, there is a deduction of only 1½ marks.

COMMUNICATION ERRORS / ERREURS DE COMMUNICATION

Shade in the circles below for a maximum total deduction of 5 marks (0.5 mark deduction per error).
Noircir les cercles ci-dessous pour une déduction maximale totale de 5 points (déduction de 0,5 point par erreur).

E1 ●	**E2** ○	**E3** ○	**E4** ○	**E5** ○
E6 ○	**E7** ●	**E8** ●	**E9** ○	**E10** ○

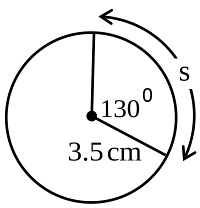
Example: Marks assigned to the student.

Marks Awarded	Booklet 1 25	Multiple Choice 7	Booklet 2 40	Communication Errors (Deduct) 1½	Total 70½
Total Marks	**36**	**9**	**45**	**maximum deduction of 5 marks**	**90**

Scoring Guidelines

Booklet 1 Questions

Use the information in the diagram to determine the value of the arc length "s".



Solution

$$130^0 \times \frac{\pi}{180^0} = \frac{13\pi}{18}$$

1 mark for conversion

$$s = \theta r$$

$$s = \frac{13\pi}{18}(3.5)$$

1 mark for substitution

2 marks

$$s = 7.941\ 248$$

$$s = 7.941 \text{ cm}$$

Exemplar 1

$S = \theta r$ $\qquad$ $3.5\text{ cm} = 0.0035$

$S = (130)(0.0035)$

$S = .455\text{ m}$

1 out of 2

+ 1 mark for substitution

Exemplar 2

$S = \theta R$

$S = (130)(3.5)$

$S = 455\text{ cm}$

1 out of 2

+ 1 mark for substitution

Solve the following equation over the interval $[0,\ 2\pi)$.

$$\tan^2\theta + 2.8\tan\theta + 1.96 = 0$$

Use the quadratic formula $x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$ for $ax^2 + bx + c = 0$.

Solution

$$\tan\theta = \frac{-2.8 \pm \sqrt{(2.8)^2 - 4(1)(1.96)}}{2(1)}$$

½ mark for substitution

$$\tan\theta = \frac{-2.8 \pm 0}{2}$$

$$\tan\theta = -1.4$$

½ mark for solving for $\tan\theta$

$$\theta_r = \tan^{-1}(1.4)$$
$$= 0.950\ 546$$

$\theta = 2.191$ $\qquad$ $\theta = 5.333$

1 mark (½ mark for each value of θ)

2 marks

Exemplar 1

$$x = \frac{-2.8 \pm \sqrt{(2.8)^2 - 4(1)(1.96)}}{2(1)}$$

$$= \frac{-2.8 \pm \sqrt{7.84 - 7.84}}{2}$$

$$= \frac{-2.8 \pm \sqrt{0}}{2}$$

$$= \frac{-2.8}{2}$$

$$\tan\theta = -1.4$$

$$= -1.4$$

$$\theta_r = 1.4$$

$$\theta = 1.742$$

$$\theta = 4.883$$

1½ out of 2

award full marks

– ½ mark for procedural error

Exemplar 2

$$\tan\theta = \frac{-2.8 \pm \sqrt{(2.8)^2 - 4(1)(1.96)}}{2(1)}$$

$$\tan\theta = \frac{-2.8 \pm \sqrt{7.84 - 7.84}}{2}$$

$$\tan\theta = -1.4$$

1 out of 2

+ ½ mark for substitution
+ ½ mark for solving for $\tan\theta$

Exemplar 3

$$\tan\theta = \frac{2.8 \pm \sqrt{(2.8)^2 - 4(1)(1.96)}}{2(1)}$$

$$\tan\theta = \frac{2.8 \pm \sqrt{0}}{2}$$

$$\tan\theta = 1.4$$

$$\theta = 0.95$$

S | A
T | C

$$\theta = 0.95$$

$$\theta = 4.09$$

2 out of 2

+ ½ mark for substitution
+ ½ mark for solving for $\tan\theta$
+ 1 mark for consistent values of θ

E6 (rounding error)
E7 (transcription error in line 1)

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Determine how many monthly investments of \$50 would have to be deposited into a savings account that pays 3% annual interest, compounded monthly, for the account's future value to be \$50,000.

Use the formula:

$$\mathrm{FV} = \frac{\mathrm{R}\left[(1+i)^n - 1\right]}{i}$$

where FV = the future value

R = the investment amount

$$i = \frac{\text{the annual interest rate}}{\text{the number of compounding periods per year}}$$

n = the number of investments

Express your answer as a whole number.

Solution

$$50\ 000 = \frac{50\left[\left(1+\frac{0.03}{12}\right)^n - 1\right]}{\frac{0.03}{12}}$$ ½ mark for substitution

$$50\ 000 = \frac{50\left[(1+0.0025)^n - 1\right]}{0.0025}$$

$$50\ 000 = 20\ 000\left(1.0025^n - 1\right)$$

$$2.5 = 1.0025^n - 1$$

$$3.5 = 1.0025^n$$ ½ mark for simplification

$$\log 3.5 = \log 1.0025^n$$ ½ mark for applying logarithms

$$\log 3.5 = n \log 1.0025$$ 1 mark for power rule

$$n = \frac{\log 3.5}{\log 1.0025}$$

$$n = 501.73$$ ½ mark for solving for n

∴ 502 monthly investments are needed.

3 marks

Exemplar 1

$FV = \$50000$
$R = \$50$
$i = 0.03$
$n = ?$

$$(0.03)\,50\,000 = \frac{50[(1+0.03)^n - 1]\,0.03}{0.03}$$

$$\frac{1500}{50} = \frac{50[(1+0.03)^n - 1]}{50}$$

$$30 = [(1.03)^n - 1$$
$$+1$$

$$31 = 1.03^n$$

$$\sqrt[1.03]{31} = n$$

$$n = 28$$

½ out of 3

+ ½ mark for simplification

Exemplar 2

$$50{,}000 = \frac{50\left[(1+.03)^n - 1\right]}{.03}$$

$$1500 = 50[(1+.03)^n - 1]$$

$$300 = (1.03)^n - 1$$

$$\log 301 = n \log 1.03$$

$$n = 193.1$$

2 out of 3

+ ½ mark for simplification
+ ½ mark for applying logarithms
+ 1 mark for power rule
+ ½ mark for solving for *n*
– ½ mark for arithmetic error in line 3
E1 (final answer not stated)

$$50\ 000 = \frac{50\left[(1+0{,}03)^n - 1\right]}{0{,}03}$$

$$\cdot 0{,}03 \quad 50\ 000 = \frac{50\left[(1{,}03)^n - 1\right]}{0{,}03} \quad \cdot 0{,}03$$

$$\frac{1500 = \cancel{50}\left[(1{,}03)^n - 1\right]}{50}$$

$$30 = (1{,}03)^n - \cancel{1}$$

$$+1 \qquad \cancel{+1}$$

$$31 = (1{,}03)^n$$

$$n = \log_{1{,}03} 31$$

$$\boxed{n = 116{,}175}$$

$$\frac{\log 31}{\log 1{,}03} = 116{,}1747752$$

2 ½ out of 3

+ ½ mark for simplification
+ ½ mark for applying logarithms
+ 1 mark for change of base
+ ½ mark for solving for *n*
E1 (final answer not stated)
E7 (notation error in line 3)

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There are 5 men and 4 women to be seated in a row.

How many arrangements are possible if two men must sit at the beginning of the row and two men must sit at the end of the row?

Solution

$$\underset{m}{\underline{5}} \cdot \underset{m}{\underline{4}} \cdot \underline{\quad 5! \quad} \cdot \underset{m}{\underline{3}} \cdot \underset{m}{\underline{2}} = 14\ 400$$

or

$$\underset{m}{\underline{5}} \cdot \underset{m}{\underline{4}} \cdot \underline{5} \cdot \underline{4} \cdot \underline{3} \cdot \underline{2} \cdot \underline{1} \cdot \underset{m}{\underline{3}} \cdot \underset{m}{\underline{2}} = 14\ 400$$

1 mark for correctly restricting men at beginning and end of the row

1 mark for 5!

2 marks

Exemplar 1

$$\overset{7}{\underline{2\cdot 1}}\ \underline{6}\ \underline{5}\ \underline{4}\ \underline{3}\ \underline{2}\ \overset{1}{\underline{2\cdot 1}} = 2!\,2!\,7! = \boxed{20\,160}$$

men men

1 out of 2

award full marks
– 1 mark for concept error of grouping

a) In the binomial expansion of $\left(\frac{3}{x^2} - 4x^5\right)^8$, determine the 3rd term.

b) In the binomial expansion of $\left(\frac{3}{x^2} - 4x^5\right)^n$, the 6th term contains x^{25}.

Solve for n.

Solution

a)

$$t_3 = {}_8C_2\left(\frac{3}{x^2}\right)^{8-2}\left(-4x^5\right)^2$$

2 marks (1 mark for ${}_8C_2$, ½ mark for each consistent factor)

$$t_3 = 28\left(\frac{3^6}{x^{12}}\right)\left(\frac{16x^{10}}{1}\right)$$

$$= \frac{28}{1} \times \frac{729}{x^{12}} \times \frac{16x^{10}}{1}$$

1 mark for simplification (½ mark for coefficient, ½ mark for exponents)

$$= \frac{326\ 592}{x^2}$$

3 marks

b)

$$x^{25} = \left(x^{-2}\right)^{n-5}\left(x^5\right)^5$$

1 mark for substitution

$$x^{25} = x^{-2n+10+25}$$

$$25 = -2n + 35$$

1 mark for equating exponents

$$-10 = -2n$$

$$5 = n$$

2 marks

Exemplar 1

a)

$$t_3 = {}_8C_2\left(\frac{3}{x^2}\right)^{8-2}(-4x^5)^2$$

$$= 28\left(\frac{3}{x^{12}}\right)(-4x^{10})$$

$$= \frac{-336}{x^2}$$

2½ out of 3

award full marks

– ½ mark for arithmetic errors in line 2

b)

$$x^{25} = (x^{-2})^{n-5}(x^5)^5$$

$$x^{25} = x^{-2n-5+25}$$

$$25 = -2n + 20$$

$$2n = -5$$

$$n = -\frac{5}{2}$$

1½ out of 2

award full marks

– ½ mark for arithmetic error in line 2

Exemplar 2

a)

$$t_3 = {}_8C_3\left(\frac{3}{x^2}\right)^{8-3}(-4x^5)^3$$
$$= 56\left(\frac{243}{x^{10}}\right)(-64x^{15})$$
$$= -870912x^5$$

2 out of 3

+ 1 mark for consistent factors
+ 1 mark for simplification

b)

$$x^{25} = (x^{-2})^{n-6}(x^5)^6$$
$$x^{25} = x^{-2n+12}\, x^{30}$$
$$25 = -2n + 12 + 30$$
$$2n = 17$$
$$n = \frac{17}{2}$$

2 out of 2

award full marks [incorrect value of *k* from a) carried over to b)]

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Question 6 R1

Given the following two functions, $f(x) = \sqrt{x-1}$ and $g(x) = x^2 + 1$, evaluate $g(f(3))$.

Solution

Method 1

$$f(3) = \sqrt{3-1}$$

$$= \sqrt{2}$$ ½ mark for $f(3)$

$$g(\sqrt{2}) = (\sqrt{2})^2 + 1$$

$$= 2 + 1$$

$$= 3$$ ½ mark for consistent value of $g(f(3))$

1 mark

Method 2

$$g(f(x)) = (\sqrt{x-1})^2 + 1$$ ½ mark for $g(f(x))$

$$= x - 1 + 1$$

$$= x$$

$$g(f(3)) = 3$$ ½ mark for evaluating $g(f(3))$

1 mark

Exemplar 1

$$f(x) = \sqrt{3-2}$$
$$= \sqrt{1}$$
$$f(x) = 1$$

so $g(x) = x^2 + 1$
$$= (1)^2 + 1$$
$$= 1 + 1$$
$$= 2$$

1 out of 1

Method 1

+ ½ mark for $f(3)$

+ ½ mark for consistent value of $g(f(3))$

E7 (transcription error in line 1)

Question 7 T2

If θ terminates in quadrant II and $\csc\theta = \frac{3}{2}$, determine the exact value of $\tan\theta$.

Solution

$$\csc\theta = \frac{r}{y}$$

$$x^2 + y^2 = r^2$$

$$x^2 + 2^2 = 3^2$$ ½ mark for identifying $y = 2$, $r = 3$

$$x^2 = 5$$

$$x = \pm\sqrt{5}$$ ½ mark for solving for x

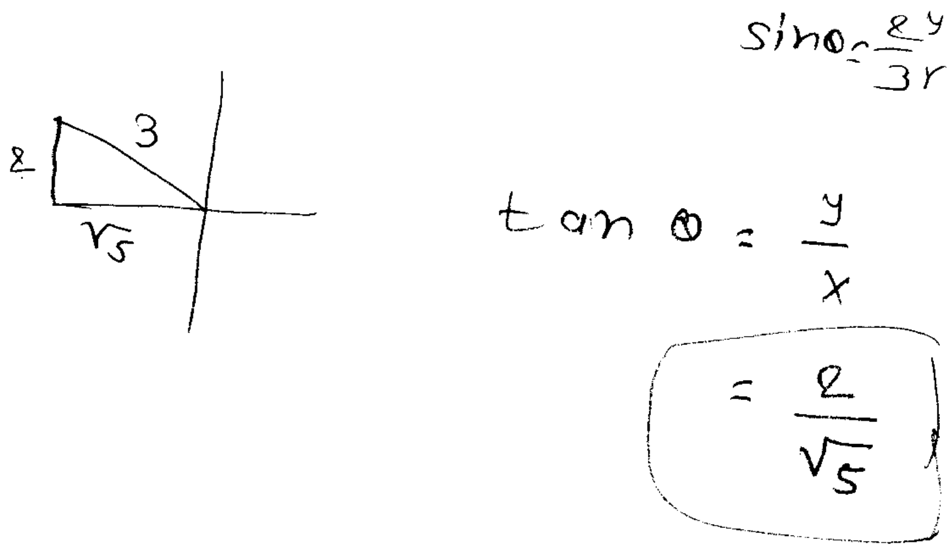
$$\tan\theta = -\frac{2}{\sqrt{5}}$$ 1 mark for $\tan\theta$ (½ mark for quadrant, ½ mark for value)

2 marks

Note(s):

§ accept any of the following values for x: $x = \pm\sqrt{5}$, $x = \sqrt{5}$, or $x = -\sqrt{5}$

Exemplar 1



1½ out of 2

+ ½ mark for indentifying $y = 2$, $r = 3$
+ ½ mark for solving for x
+ ½ mark for the value of $\tan\theta$

a) Determine the remainder when $x^4 - 3x^2 + 1$ is divided by $x + 2$.

b) Is $x + 2$ a factor of $x^4 - 3x^2 + 1$? Explain your reasoning.

Solution

a) $(-2)^4 - 3(-2)^2 + 1$

$16 - 12 + 1$

5

1 mark for remainder theorem

or

–2	1	0	–3	0	1
		–2	4	–2	4
	1	–2	1	–2	5

or

1 mark for synthetic division

1 mark

The remainder is 5.

b) For $x + 2$ to be a factor of $x^4 - 3x^2 + 1$, the remainder must be 0. Since the remainder is 5, $x + 2$ is not a factor.

1 mark for explanation

1 mark

Exemplar 1

a)

$x + 2 = 0$

$x = -2$

-2	1	0	-3	1
+		-2	4	-2
	1	-2	1	-1

remainder = -1

½ out of 1

award full marks
– ½ mark for procedural error in setup

b)

No since there is a remainder it is not a factor.

1 out of 1

Exemplar 2

a)

-2	1	$+0$	-3	$+0$	$+1$
	$\downarrow$	-2	4	-2	-8
	1	-2	1	4	$\boxed{-7}$

$x^3 - 2x^2 + x + 4$ remainder of -7

½ out of 1

+ 1 mark for synthetic division
– ½ mark for arithmetic error

b)

$x = -2$ $\quad (-2)^4 - 3(2)^2 + 1 = 0$

$16 - 12 + 1 = 5 \neq 0$

$4 + 1 = 5 \neq 0$

No it is not

0 out of 1

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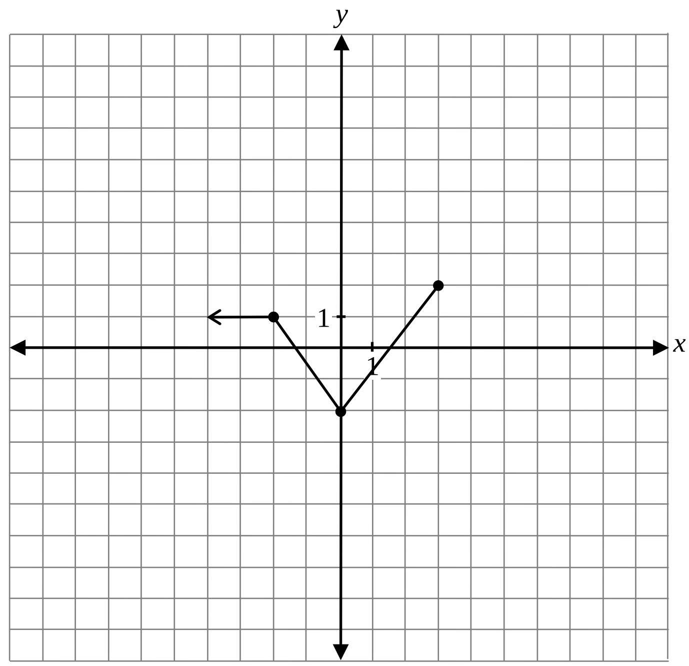
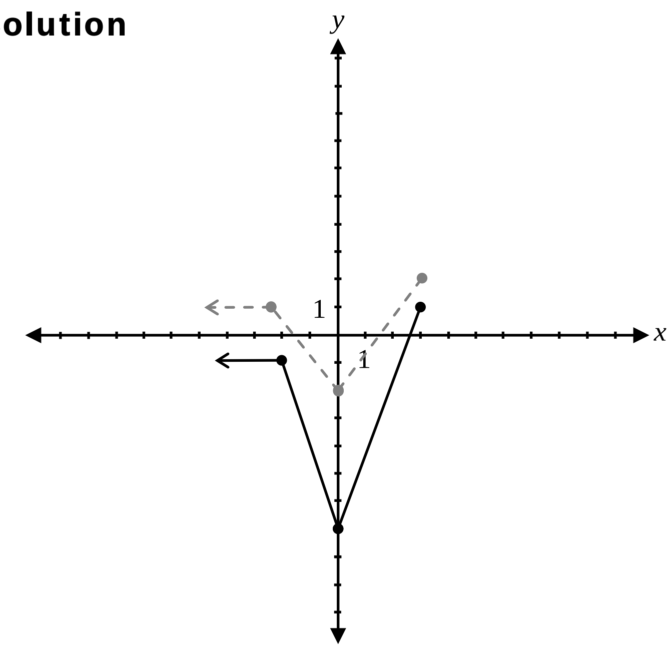
Given the graph of $y = f(x)$ below, sketch the graph of $y = 2f(x) - 3$.



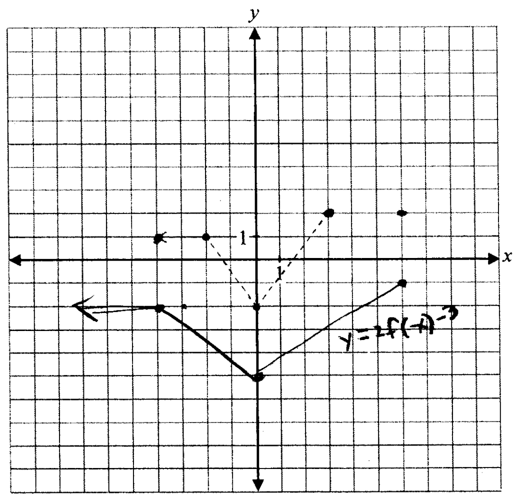
Solution



1 mark for vertical stretch
1 mark for vertical shift

2 marks

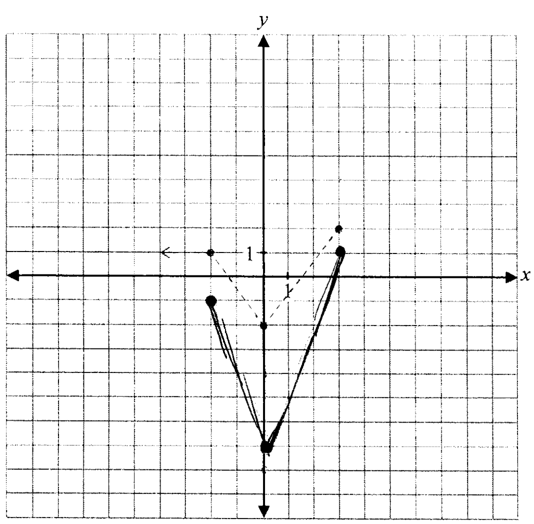
Exemplar 1



1 out of 2

+ 1 mark for vertical shift

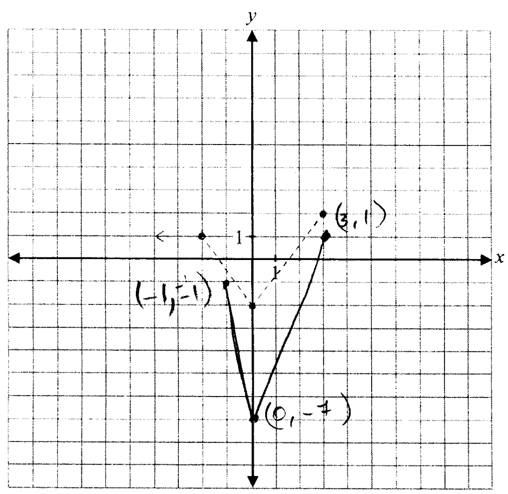
Exemplar 2



1½ out of 2

+ 1 mark for vertical stretch
+ 1 mark for vertical shift
– ½ mark for incorrect shape (left side)

Exemplar 3



1 ½ out of 2

+ 1 mark for vertical stretch
+ 1 mark for vertical shift
− ½ mark for incorrect shape (left side)
E7 (transcription error for point at $x = -1$ instead of $x = -2$)

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Determine one possible restriction for the domain of $f(x) = (x-1)^2$ so that the inverse of $f(x)$ is a function.

Solution

$x \geq 1$

or

$x < 1$

1 mark

Note(s):

§ Many solutions are possible. Any solutions which restrict $f(x)$ to a one-to-one function are correct.

Exemplar 1

$x \geq 0$

0 out of 1

Exemplar 2

$x \geq 2$

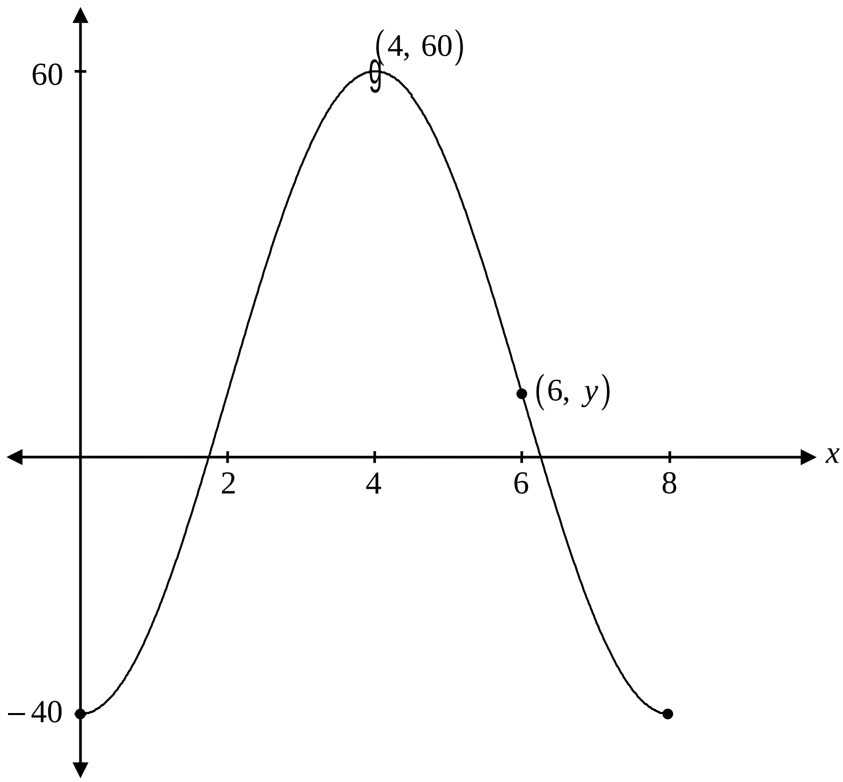
1 out of 1

Exemplar 3

$x \neq 1$

0 out of 1

Using the graph of the sinusoidal function below, find the value of y in the point $(6,\ y)$.

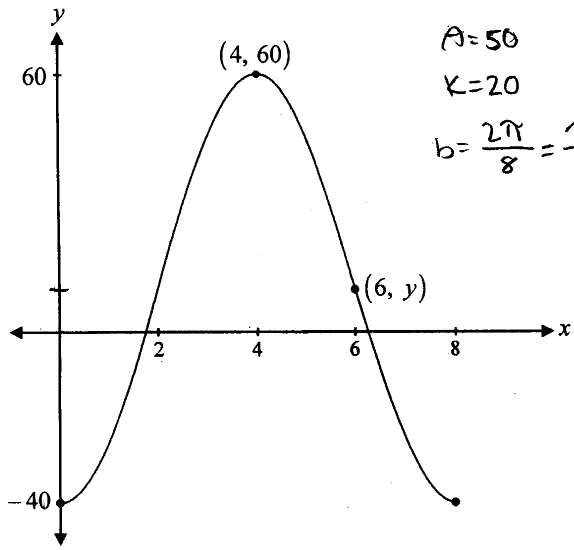


Solution

$y = 10$

1 mark

Exemplar 1



$A = 50$

$k = 20$

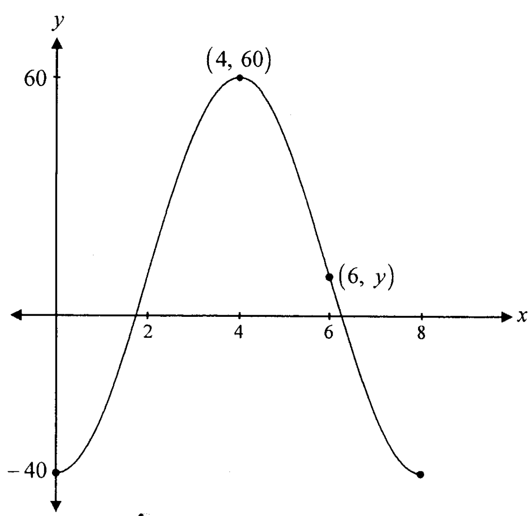
$b = \frac{2\pi}{8} = \frac{\pi}{4}$

$y = 50\sin[$

$y = 20$

0 out of 1

Exemplar 2



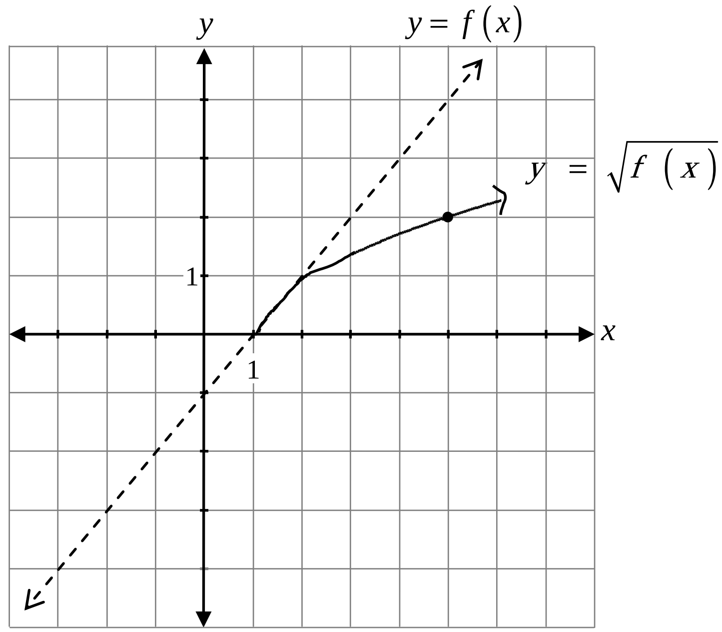
(6, 20) $y = 20$

0 out of 1

Billy was given the graph of $y = f(x)$.

He was asked to sketch the graph of $y = \sqrt{f(x)}$.

His answer is given on the graph below.



Explain the error Billy made when sketching the graph of $y = \sqrt{f(x)}$.

Solution

Billy's graph should be above the line $y = f(x)$ over the interval where x is between 1 and 2.

1 mark

Exemplar 1

He didn't sketch the assymptotes
and the point doesn't start at (0,1) but 0,0

0 out of 1

Exemplar 2

The change applied on the graph of $y = f(x)$ is
$(x, y) \longrightarrow (x, \sqrt{y})$ then ~~if~~ we pick a point from
the graph of $y = f(x)$ (2, 1) the point remain
the same when we apply the change so, the
graph of $y = \sqrt{f(x)}$ should pass the point (2,1) but
Billy's graph of $y = \sqrt{f(x)}$ didn't pass with the
point (2,1)

0 out of 1

Exemplar 3

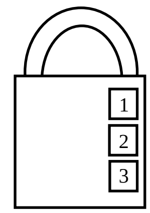
Part of Billy's graph should be above y=f(x).

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Question 13 P2, P3

Explain why a locker combination should really be called a locker permutation.



Solution

The order of the locker combination matters; therefore, it is a permutation.

1 mark

Exemplar 1

After using the first no., you can't use it again. This changes the no. of options left. Therefore, it is a permutation. not combination.

0 out of 1

Question 14 R5

The graph of $f(x) = x^2 + 4$ is reflected over the x-axis.
Write the equation of the new function.

$y =$ ______________________

Solution

$y = -x^2 - 4$

1 mark

Exemplar 1

$y = -x^2 + 4$

0 out of 1

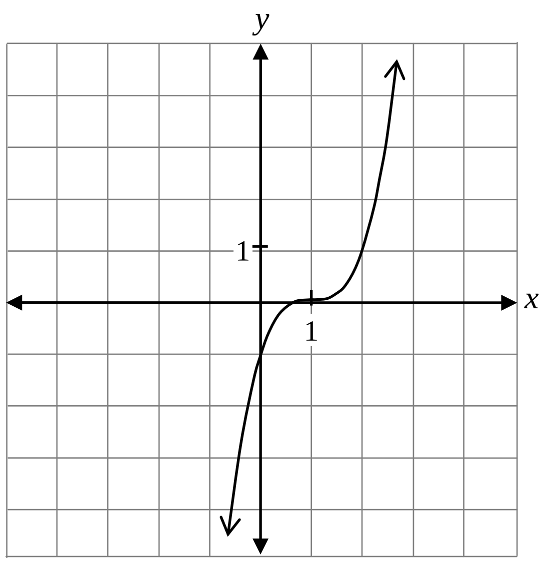
Exemplar 2

$y = -f(x)$

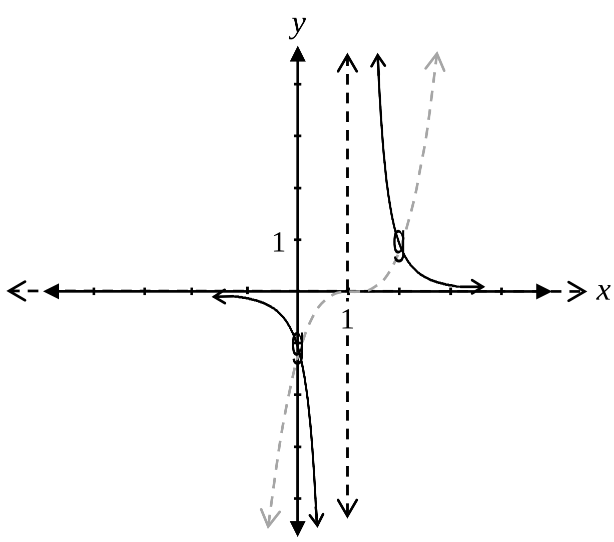
1 out of 1

Question 15 R1

Given the graph of $y = f(x)$ below, sketch the graph of $y = \frac{1}{f(x)}$.



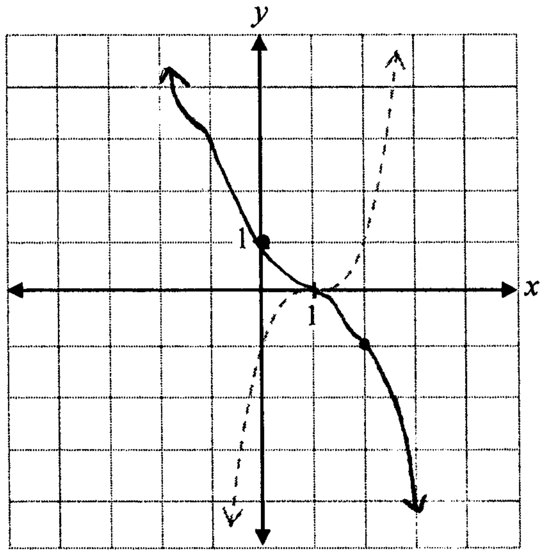
Solution



1 mark for vertical asymptote at $x = 1$
½ mark for graph left of vertical asymptote
½ mark for graph right of vertical asymptote

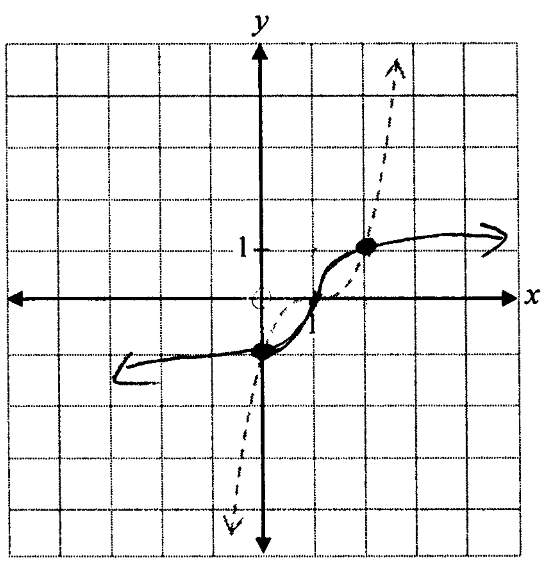
2 marks

Exemplar 1



0 out of 2

Exemplar 2



0 out of 2

Divide $\left(x^3 - 5x - 4\right)$ by $(x + 1)$.

Solution

Method 1

–1	1	0	–5	–4
		–1	1	4
	1	–1	–4	0

1 mark for set-up of synthetic division using addition

$x^2 - x - 4$

1 mark for quotient

2 marks

Method 2

$$
\begin{array}{r}
x^2 - x - 4 \\
x+1\overline{)\,x^3 + 0x^2 - 5x - 4} \\
\underline{-\left(x^3 + x^2\right)} \qquad\quad \\
-x^2 - 5x \quad\;\; \\
\underline{-\left(-x^2 - x\right)} \quad\;\; \\
-4x - 4 \\
\underline{-\left(-4x - 4\right)} \\
0
\end{array}
$$

1 mark for set-up of long division
1 mark for quotient

2 marks

Method 3

1	1	0	–5	–4
		1	–1	–4
	1	–1	–4	0

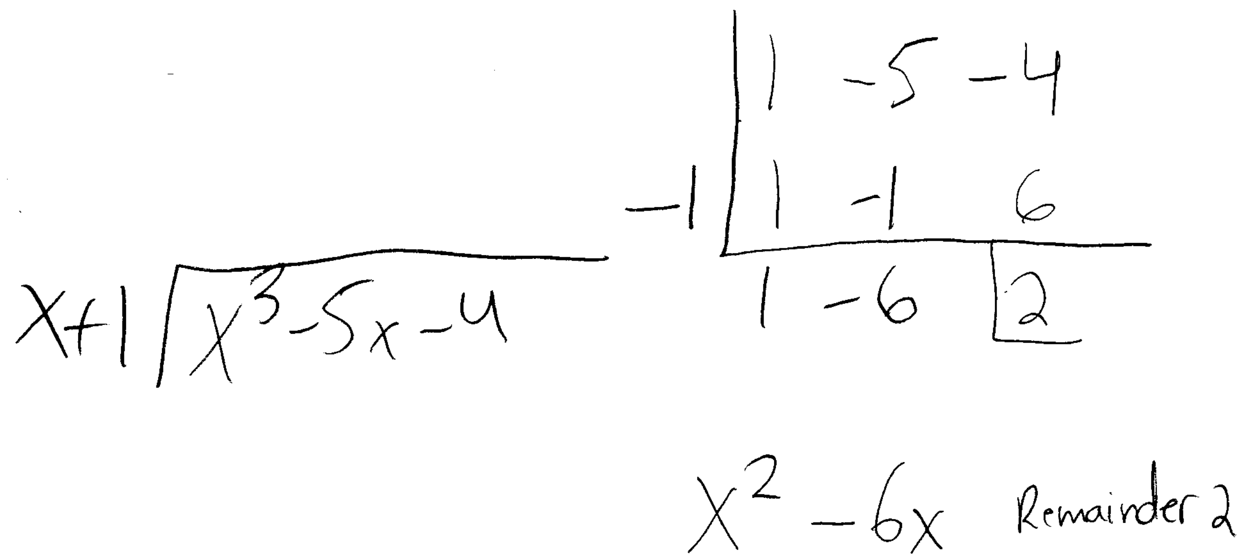
1 mark for set-up of synthetic division using subtraction

$x^2 - x - 4$

1 mark for quotient

2 marks

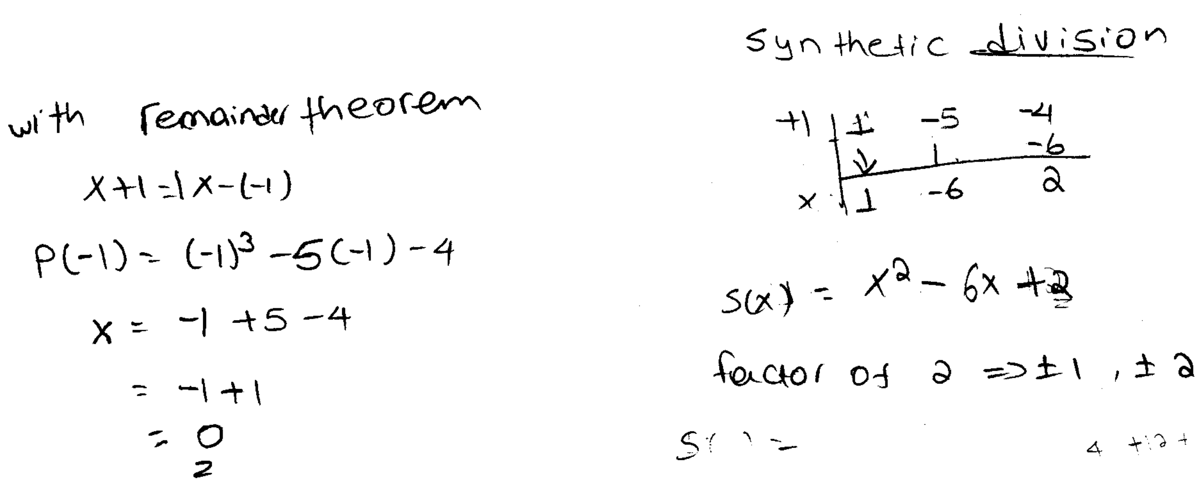
Exemplar 1



1 out of 2

+ 1 mark for quotient consistent with error in set-up

Exemplar 2



1 out of 2

+ 1 mark for quotient consistent with error in set-up

Question 17 P4

You are given the following row of Pascal's Triangle.

1	7	21	35	35	21	7	1

Determine the values of the next row.

Solution

1	8	28	56	70	56	28	8	1

1 mark

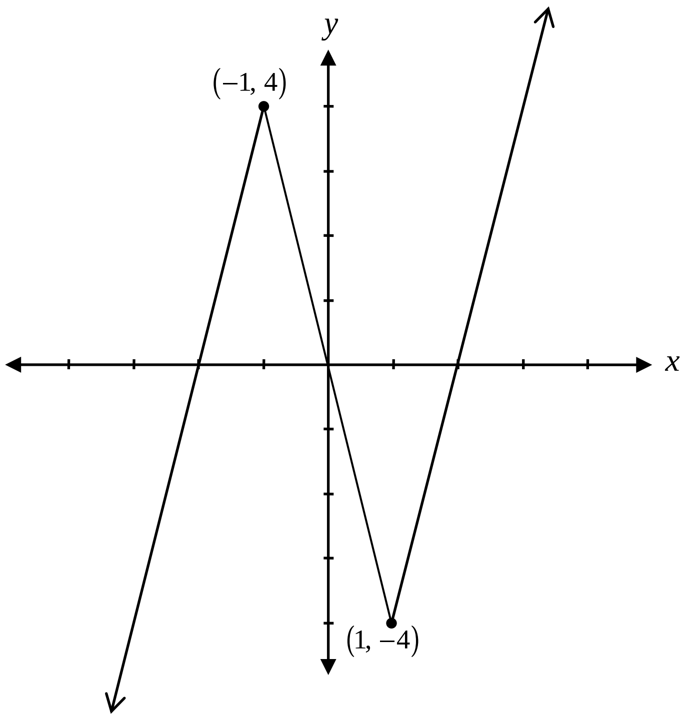
Exemplar 1

1 8 28 56 80 56 28 8 1

½ out of 1

award full marks
– ½ mark for arithmetic error

Given the graph of $y = f(x)$ below, state the domain and range of $y = \sqrt{f(x)}$.



Solution

Domain: $\{x \in \mathbb{R} \mid -2 \le x \le 0 \text{ or } x \ge 2\}$

or

$[-2, 0] \cup [2, \infty)$

1 mark for domain (½ mark for $-2 \le x \le 0$, ½ mark for $x \ge 2$)

Range: $\{y \in \mathbb{R} \mid 0 \le y\}$

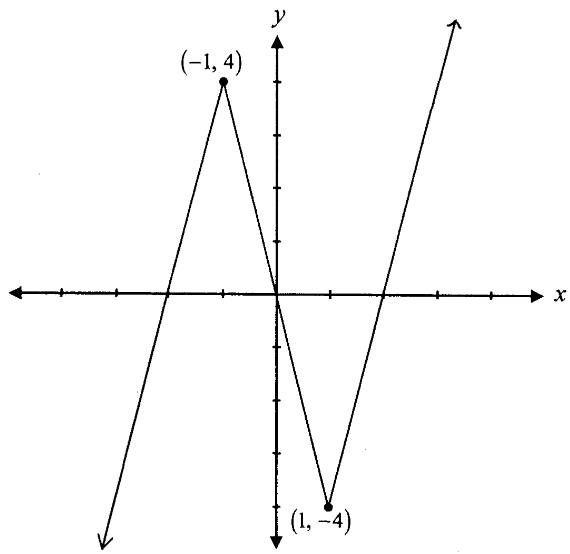
or

$[0, \infty)$

1 mark for range

2 marks

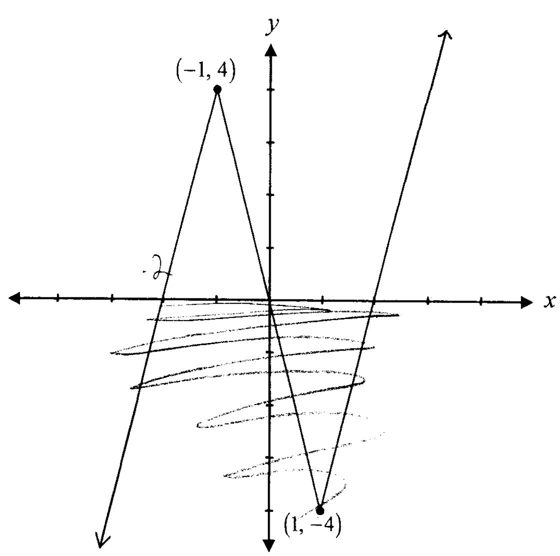
Exemplar 1



x|x∈R
y|y∈R

0 out of 2

Exemplar 2



R=[0, 4]
D= [-2, 0) ∪ [2, ∞)

1 out of 2

+ 1 mark for domain
E8 (bracket error made when stating domain in line 2)

Prove the identity below for all permissible values of θ:

$$\frac{1-\tan^2\theta}{1+\tan^2\theta}=\cos 2\theta$$

Solution

Method 1

Left-Hand Side	Right-Hand Side
$\dfrac{1-\tan^2\theta}{1+\tan^2\theta}$	$\cos 2\theta$
$\dfrac{1-\tan^2\theta}{\sec^2\theta}$	
$\dfrac{1}{\sec^2\theta}-\dfrac{\tan^2\theta}{\sec^2\theta}$	
$\cos^2\theta-\dfrac{\frac{\sin^2\theta}{\cos^2\theta}}{\frac{1}{\cos^2\theta}}$	
$\cos^2\theta-\sin^2\theta$	
$\cos 2\theta$	

1 mark for correct substitution of appropriate identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Method 2

Left-Hand Side	Right-Hand Side
$\dfrac{1-\tan^2\theta}{1+\tan^2\theta}$	$\cos 2\theta$
$\dfrac{1-\dfrac{\sin^2\theta}{\cos^2\theta}}{1+\dfrac{\sin^2\theta}{\cos^2\theta}}$	
$\dfrac{\dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta}}{\dfrac{\cos^2\theta+\sin^2\theta}{\cos^2\theta}}$	
$\dfrac{\cos^2\theta-\sin^2\theta}{\cos^2\theta+\sin^2\theta}$	
$\cos^2\theta-\sin^2\theta$	
$\cos 2\theta$	

1 mark for correct substitution of appropriate identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Method 3

Left-Hand Side	Right-Hand Side
$\dfrac{1-\tan^2\theta}{1+\tan^2\theta}$	$\cos 2\theta$
$\dfrac{1-\left(\sec^2\theta-1\right)}{\sec^2\theta}$	
$\dfrac{2-\sec^2\theta}{\sec^2\theta}$	
$\dfrac{2-\dfrac{1}{\cos^2\theta}}{\dfrac{1}{\cos^2\theta}}$	
$\dfrac{\dfrac{2\cos^2\theta-1}{\cos^2\theta}}{\dfrac{1}{\cos^2\theta}}$	
$2\cos^2\theta-1$	
$\cos 2\theta$	

1 mark for correct substitution of appropriate identities
1 mark for algebraic strategies
1 mark for logical process to prove the identity

3 marks

Left-Hand Side	Right-Hand Side
$\dfrac{1-\tan^2\theta}{1+\tan^2\theta}$	$\cos 2\theta$
$\dfrac{1-\tan^2\theta}{\sec^2\theta}$	$= 1-2\sin^2\theta$
$1-\tan^2\theta \cdot \dfrac{\cos^2\theta}{1}$	$= 2\cos^2\theta$
$= 1-\tan^2\theta\cos^2\theta$	
$= 1-\dfrac{\sin^2\theta}{\cancel{\cos^2\theta}}\cdot\cancel{\cos^2\theta}$	
$= 1-\sin^2\theta$	
$= \cos^2\theta$	

2 out of 3

+ 1 mark for correct substitution of appropriate identities
+ 1 mark for logical process to prove the identity

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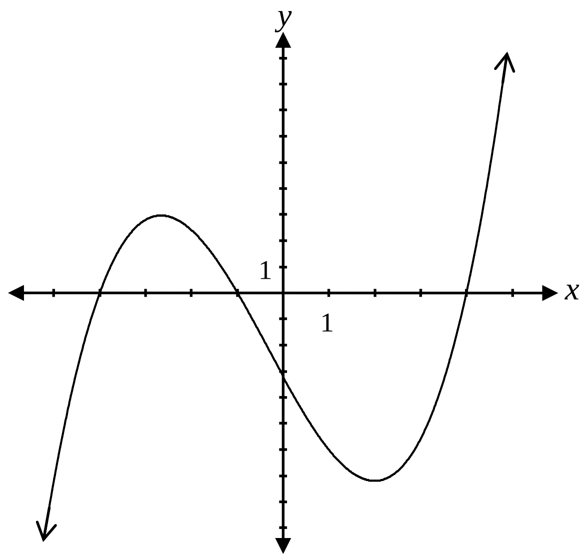
Booklet 2 Questions

Answer Key for Multiple-Choice Questions

Question	Answer	Learning Outcome
20	C	R1
21	A	R8
22	B	R2
23	D	R13
24	A	R14
25	C	T5
26	A	T4
27	B	R7
28	D	R12
29	B	R5

Question 20 R1

Given the graph of the function of $f(x)$ below, what is the range of $y = |f(x)|$?



a) $y \in$ ¡

b) $y \geq -7$

c) $y \geq 0$

d) $-4 \leq y \leq -1$ or $y \geq 4$

Question 21 R8

Simplify the following expression:

$$\frac{1}{2}\log_a 36 - \log_a 2$$

a) $\log_a 3$

b) $\log_a 4$

c) $\log_a 9$

d) $\log_a 12$

Question 22 R2

Given $f(x) = x^2 - x + 2$, an equation that represents the graph of $f(x)$ shifted 3 units to the right is:

a) $y = (x+3)^2 - (x+3) - 3$

b) $y = (x-3)^2 - (x-3) + 2$

c) $y = (x-3)^2 - x - 2$

d) $y = x^2 - x + 2 - 3$

Question 23 R13

What is the domain of the function $y = \sqrt{-4x}$?

a) $\{x \in ¡ \mid x \geq 2\}$

b) $\{x \in ¡ \mid x \leq 2\}$

c) $\{x \in ¡ \mid x \geq 0\}$

d) $\{x \in ¡ \mid x \leq 0\}$

Question 24 R14

Which of the following is true about the two functions below?

$$f(x) = \frac{(x+2)(x-2)}{x-2} \qquad g(x) = \frac{(x-2)(x+1)}{(x+2)(x-2)}$$

a) Both have a point of discontinuity (hole) when $x = 2$.

b) Both have the same vertical asymptote.

c) Both have the same horizontal asymptote.

d) Both have the same y-intercept.

Question 25 T5

The general solution to the equation $\cos\theta = -\frac{1}{2}$ is:

a) $\left.\begin{aligned}\theta &= \frac{\pi}{3} + 2\pi k\\ \theta &= \frac{5\pi}{3} + 2\pi k\end{aligned}\right\}$ where $k \in \mathrm{I}$

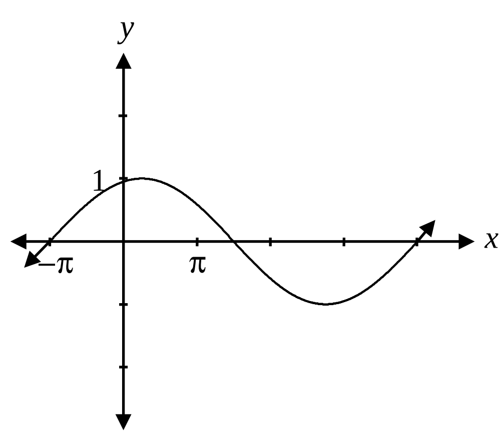
b) $\left.\begin{aligned}\theta &= \frac{\pi}{3} + \pi k\\ \theta &= \frac{5\pi}{3} + \pi k\end{aligned}\right\}$ where $k \in \mathrm{I}$

c) $\left.\begin{aligned}\theta &= \frac{2\pi}{3} + 2\pi k\\ \theta &= \frac{4\pi}{3} + 2\pi k\end{aligned}\right\}$ where $k \in \mathrm{I}$

d) $\left.\begin{aligned}\theta &= \frac{2\pi}{3} + \pi k\\ \theta &= \frac{4\pi}{3} + \pi k\end{aligned}\right\}$ where $k \in \mathrm{I}$

Question 26 T4

If the equation $y = \sin\left(b\left(x + \pi\right)\right)$ is represented by the following graph, what is the value of b?



a) $\frac{2}{5}$ b) $\frac{5}{2}$ c) $\frac{2\pi}{5}$ d) 5π

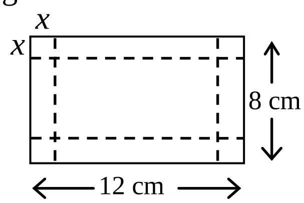
Question 27 R7

Which of the following is closest to the value of $\log_2 40 + \log_5 125$?

a) 3 b) 8 c) 10 d) 45

Question 28 R12

A sheet of paper 12 cm long and 8 cm wide is used to make a box with no lid. Equal squares of side length x cm are cut from each of the corners and the sides are folded up to make the box.



Which of the following expresses the volume of the box?

a) $V(x) = x(12 + x)(8 + x)$

b) $V(x) = x(12 - x)(8 - x)$

c) $V(x) = x(12 + 2x)(8 + 2x)$

d) $V(x) = x(12 - 2x)(8 - 2x)$

Question 29 R5

Given that the graph of $f(x)$ contains the point $(-3, 5)$, what point must be on the graph of $f(-x)$?

a) $(-3, -5)$

b) $(3, 5)$

c) $(3, -5)$

d) $(5, -3)$

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Question 30 T1

Determine one positive and one negative coterminal angle with the angle $\frac{5\pi}{6}$.

Solution

$\frac{17\pi}{6}$ and $-\frac{7\pi}{6}$

½ mark for one positive coterminal angle
½ mark for one negative coterminal angle

1 mark

Note(s):

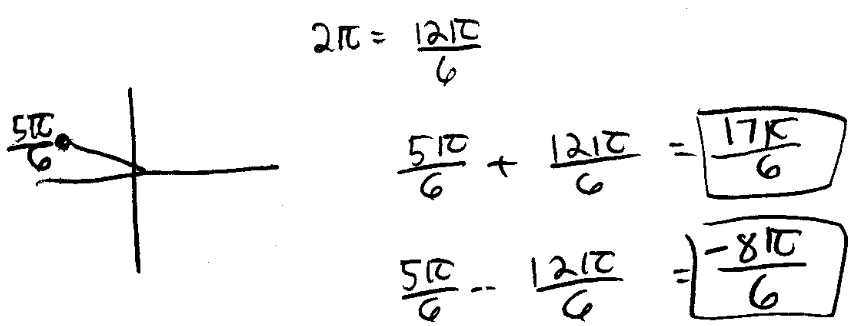
- Other answers are possible.
- Answers in degrees are acceptable.

Exemplar 1

½ out of 1

+ ½ mark for one negative coterminal angle

Exemplar 2



½ out of 1

+ ½ mark for one positive coterminal angle

Evaluate:

$$\left(\sin\frac{11\pi}{3}\right)\left(\sec\frac{11\pi}{6}\right)$$

Solution

$$\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{2}{\sqrt{3}}\right) = -1$$

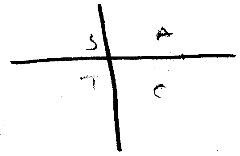
1 mark for $\sin\frac{11\pi}{3}$ (½ mark for quadrant, ½ mark for value)

1 mark for $\sec\frac{11\pi}{6}$ (½ mark for quadrant, ½ mark for value)

2 marks

Exemplar 1

$$-\frac{\sqrt{3}}{2} \quad + \frac{2}{\sqrt{3}}$$



$$-\frac{\sqrt{3}}{2} + \frac{2}{\sqrt{3}}$$

$$\frac{3+4}{2\sqrt{3}} \qquad \frac{7}{2\sqrt{3}}$$

1 out of 2

award full marks
– ½ mark for procedural error in line 2
– ½ mark for arithmetic error in line 3

Exemplar 2

$$\sec = \frac{1}{\cos}$$

$$\frac{1}{\cos} = \frac{11\pi}{6}$$

$$\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{2}{(\sqrt{3})}\right)$$

$$\frac{6}{\cos} = \frac{11\pi}{1}$$

$$\cos = \frac{6}{11\pi}$$

$$\frac{2\sqrt{3}}{2\sqrt{3}} = \boxed{1}$$

$$\frac{1}{\cos} = \frac{11\pi}{6}$$

1½ out of 2

award full marks
– ½ mark for arithmetic error in line 2
E3 (variable omitted on right-hand side)

Question 32 T5

Given the equation $2\sin^2\theta - 3\sin\theta + 1 = 0$, verify that $\theta = \frac{\pi}{2}$ is a solution.

Solution

$$\begin{aligned}\text{Left-hand side} &= 2\left(\sin\frac{\pi}{2}\right)^2 - 3\left(\sin\frac{\pi}{2}\right) + 1 \\ &= 2(1)^2 - 3(1) + 1 \\ &= 0 \\ &= \text{Right-hand side}\end{aligned}$$

1 mark for verification

1 mark

Exemplar 1

$$2\sin^2\left(\frac{\pi}{2}\right) - 3\sin\left(\frac{\pi}{2}\right) = 0$$

$$2 - 3 = 0$$

$$2 - 3 \neq 0$$

$$-1 \neq 0$$

1 out of 1

E7 (transcription error in line 1)

Exemplar 2

$$(2\sin\theta - 1)(\sin\theta - 1)$$

$$\sin\theta = \frac{1}{2} \qquad \sin\theta = 1$$

$$\frac{\pi}{6} \qquad \frac{\pi}{2}$$

1 out of 1

E2 (changing an equation to an expression)

$$(2\sin^2\theta + 2\sin\theta) + (1\sin\theta + 1)$$

$$2\sin\theta(\sin\theta + 1) + 1(\sin\theta + 1)$$

$$(2\sin\theta + 1)(\sin\theta + 1)$$

$$\sin\theta = -1/2 \qquad \sin\theta = -1 \qquad \theta \neq \frac{\pi}{2}$$

$$\theta = \frac{7\pi}{6} \qquad \theta = \frac{11\pi}{6} \qquad \theta = \frac{3\pi}{2}$$

1 out of 1

E2 (changing an equation to an expression)
E7 (transcription error in line 1)

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Question 33 R8

Using the laws of logarithms, expand:

$$\log_a\left(\frac{x \cdot y}{z}\right)$$

Solution

$\log_a x + \log_a y - \log_a z$

1 mark for product rule
1 mark for quotient rule

2 marks

Exemplar 1

$$\log_a(x + y - z)$$

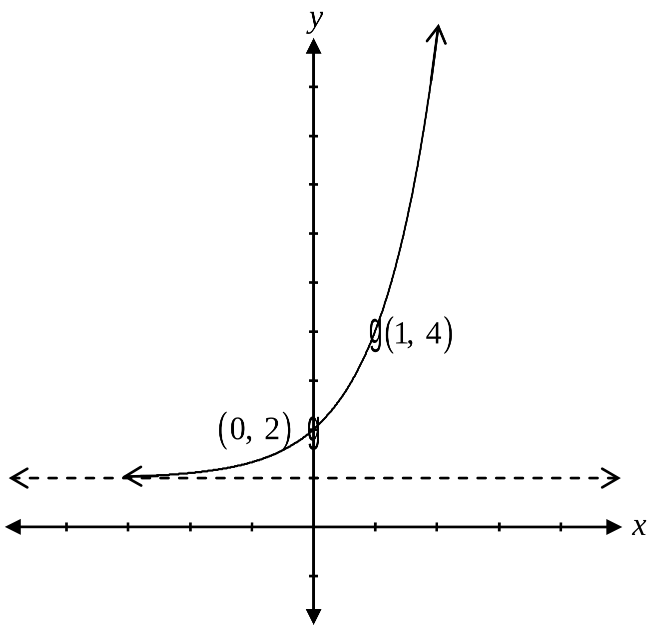
0 out of 2

a) Sketch the graph of $f(x) = 3^x + 1$.

b) Sketch the graph of $f^{-1}(x)$.

Solution

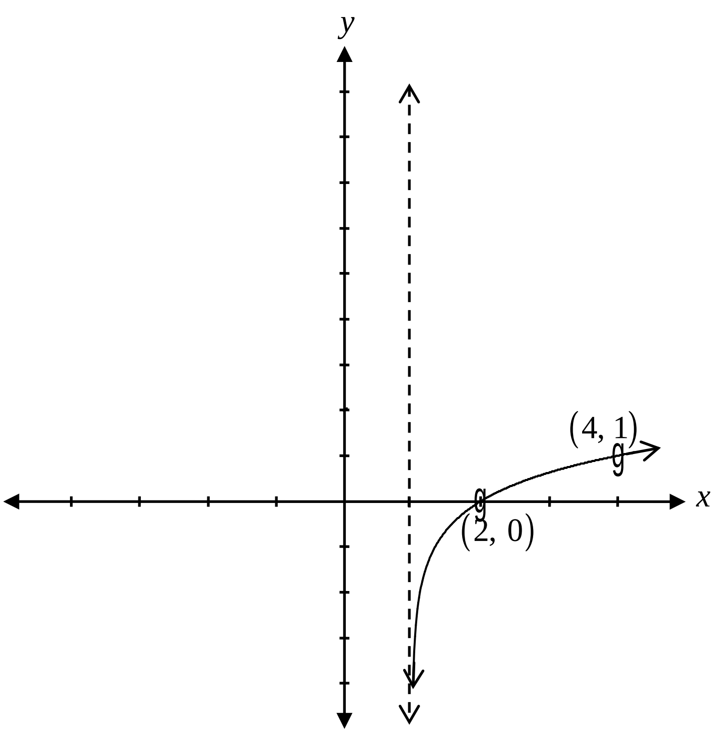
a)



½ mark for increasing exponential function
½ mark for y-intercept at $(0, 2)$
½ mark for asymptote at $y = 1$
½ mark for consistent point on exponential function

2 marks

b)

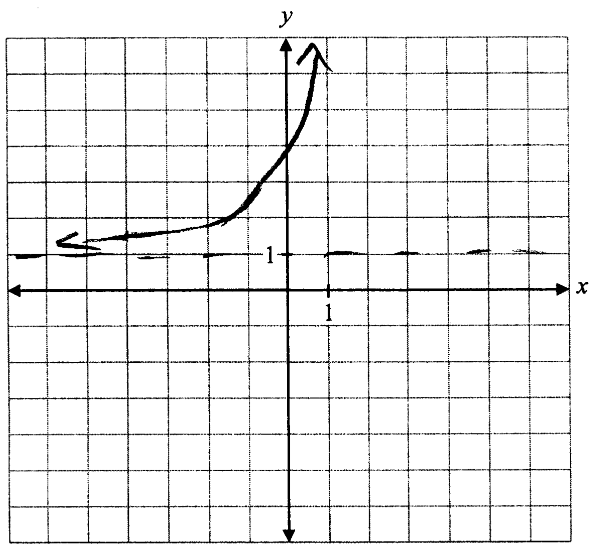


1 mark for consistent graph of the inverse

1 mark

Exemplar 1

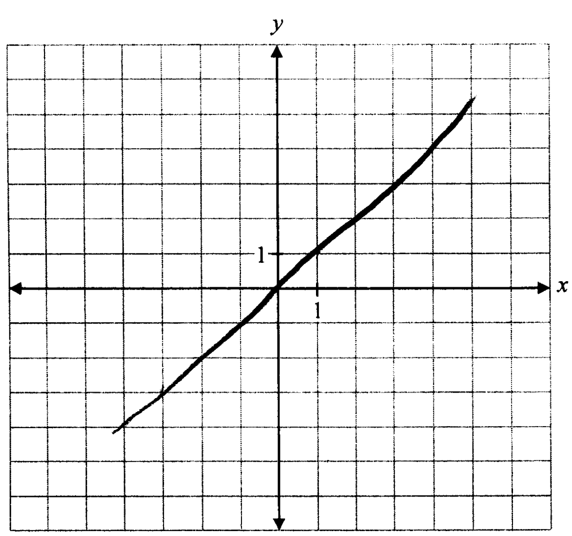
a)



1 out of 2

+ ½ mark for increasing exponential function
+ ½ mark for asymptote at $y = 1$

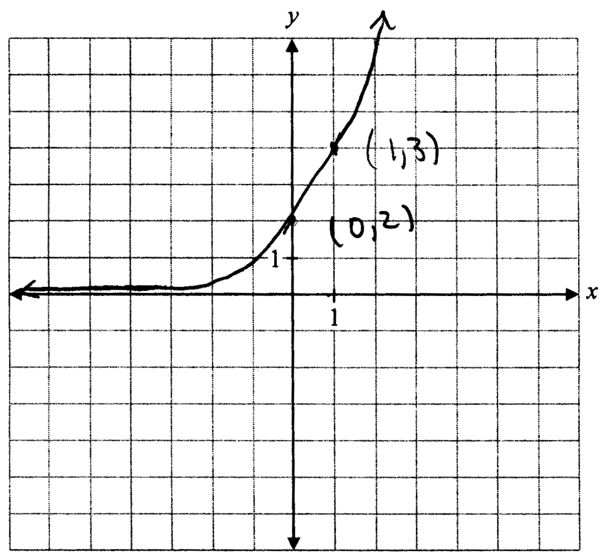
b)



0 out of 1

Exemplar 2

a)

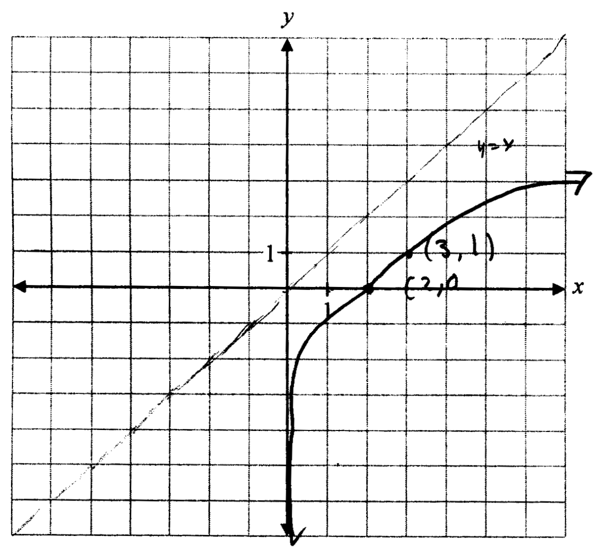


1 out of 2

+ ½ mark for increasing exponential function

+ ½ mark for *y*-intercept at $\left(0,\ 2\right)$

b)

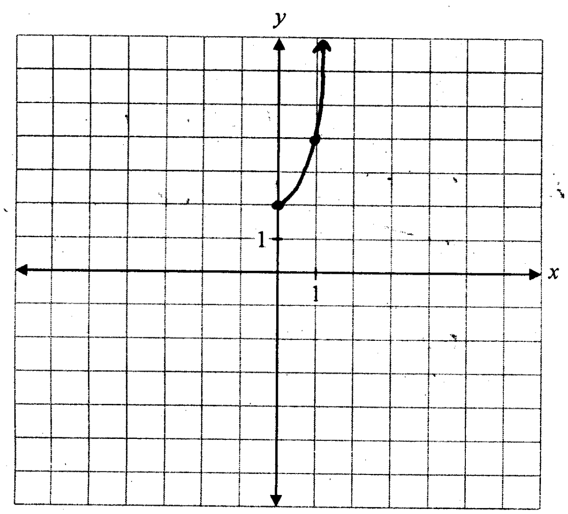


1 out of 1

+ 1 mark for consistent graph of the inverse

Exemplar 3

a)

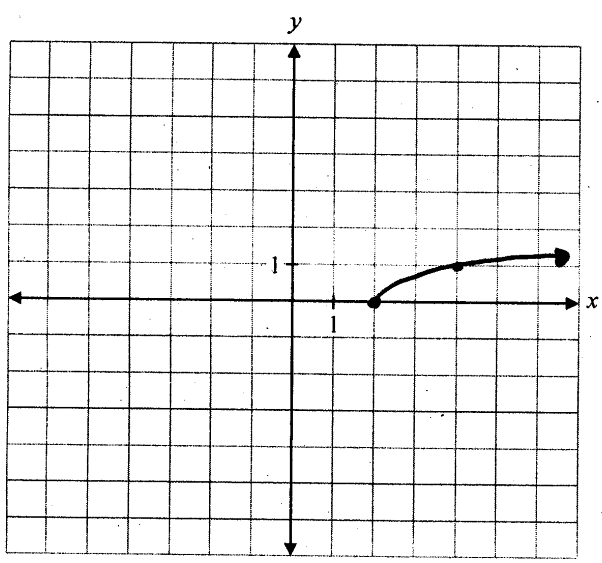


1 out of 2

+ ½ mark for y-intercept at $(0, 2)$

+ ½ mark for consistent point on exponential function

b)



1 out of 1

+ 1 mark for consistent graph of the inverse

Question 35 R9

Determine the x-intercept and y-intercept of $y = \log_2(x+4) - 1$.

Solution

Substitute x with 0.

$$y = \log_2 4 - 1$$

$$y = 2 - 1$$ ½ mark for evaluating logarithm

$$y = 1$$ ½ mark for consistent value of y

$\therefore$ y-intercept is 1

Substitute y with 0.

$$0 = \log_2(x + 4) - 1$$

$$1 = \log_2(x + 4)$$

$$2 = x + 4$$ ½ mark for exponential form

$$-2 = x$$ ½ mark for consistent value of x

$\therefore$ x-intercept is -2

2 marks

Note(s):

§ award ½ mark if student substitutes x with 0 to find the y-intercept and y with 0 to find the x-intercept

Exemplar 1

$0 = \log_2(x+4) - 1$

$1 = \log_2(x+1)$

$2 = x+1$

$x = 1$

x-int = 1

$y = \log_2(4) - 1$

$y = 2 - 1$

$y = 1$

y-int = 1

2 out of 2

E7 (transcription error in line 2, left branch)

Exemplar 2

x-int = −2

y-int = 1

2 out of 2

Exemplar 3

$y = \log_2(0+4)-1$
$y = \log_2(4)-1$
$4 = 2^y - 1$
$y=1$

$0=\log_2(x+4)+1$
$(x+4) = 2^0$
$-4 \quad -4$
$x = 1-4$
$x = -3$

½ out of 2

+ ½ mark for substituting *x* with 0 to find the *y*-intercept and *y* with 0 to find the *x*-intercept

Note(s):

§ award ½ mark if student substitutes *x* with 0 to find the *y*-intercept and *y* with 0 to find the *x*-intercept

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Explain the error that was made when solving the following equation:

$\sin 2\theta = \cos\theta,$ where $\theta \in$ ¡

$$\sin 2\theta = \cos\theta$$
$$2\sin\theta\cos\theta = \cos\theta$$
$$\frac{2\sin\theta\cos\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta}$$
$$2\sin\theta = 1$$
$$\sin\theta = \frac{1}{2}$$
$$\theta = \frac{\pi}{6} + 2k\pi,\ \frac{5\pi}{6} + 2k\pi,\ k \in I$$

Solution

The student divided by $\cos\theta$ instead of factoring out $\cos\theta$.

1 mark

or

There are two more solutions that come from the equation $\cos\theta = 0$.

or

The student cannot divide both sides by $\cos\theta$ since $\cos\theta$ could equal 0.

Exemplar 1

$\sin\theta = \frac{\pi}{6}$

You cannot divide by $\cos\theta$.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 2

You can not divide one side by the other because it may be a zero which would make it undefined.

½ out of 1

award full marks
– ½ mark for lack of clarity in explanation

Exemplar 3

$$\sin 2\theta = \cos\theta$$

$$2\sin\theta\cos\theta = \cos\theta$$

$$2\sin\theta\cos\theta - \cos\theta = 0$$

$$\cos\theta(2\sin\theta - 1) = 0$$

$$\cos\theta = 0 \qquad \sin\theta = 1/2$$

$$\theta = \frac{\pi}{2} + 2k\pi, \ \frac{3\pi}{2} + 2k\pi, \ \frac{\pi}{6} + 2k\pi, \ \frac{5\pi}{6} + 2k\pi, \quad k \in I.$$

0 out of 1

Given $f(x) = x^2 - 2x - 3$ and $g(x) = x + 1$:

a) Write the equation of $y = f(g(x))$.

b) Sketch the graph of $y = f(g(x))$.

Solution

a) $f(g(x)) = (x+1)^2 - 2(x+1) - 3$

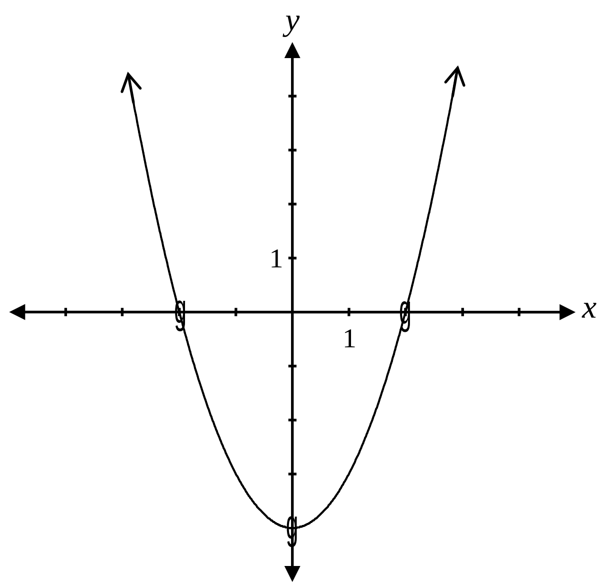
$= x^2 + 2x + 1 - 2x - 2 - 3$

$= x^2 - 4$

1 mark for composition

1 mark

b)



1 mark for graph (½ mark for *x*-intercepts, ½ mark for *y*-intercept)

1 mark

Exemplar 1

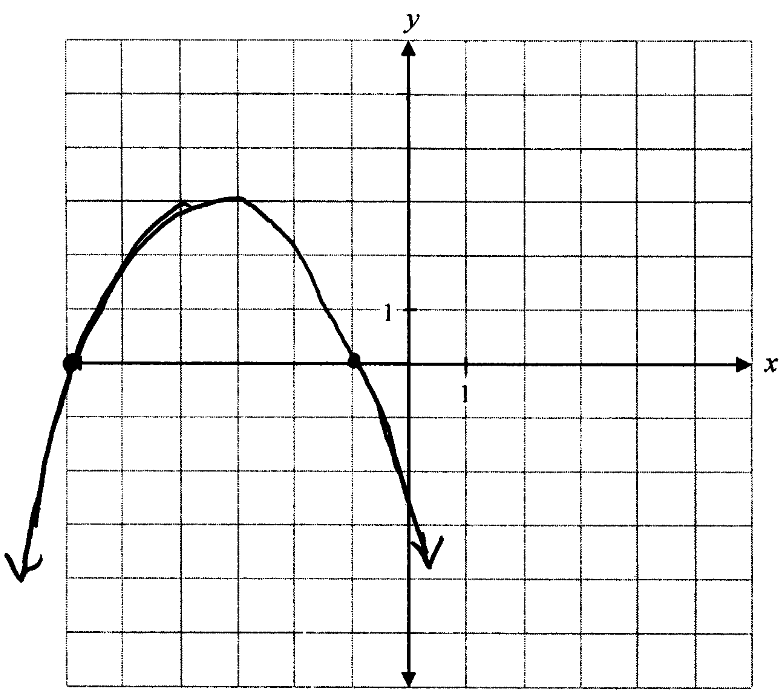
a)

$$f[g(x)] = (x+1)^2 - 2(x+1) - 3$$
$$= (x+1)(x+1) - 2x + 2 - 3$$
$$= x^2 + 4x + 4x + 1 - 2x + 2 - 3$$
$$= x^2 + 6x$$
$$= x(x+6)$$
$$= x + 1 = 0$$
$$x = -1, -6$$

½ out of 1

+ 1 mark for composition
– ½ mark for arithmetic error in line 3

b)



0 out of 1

Exemplar 2

a)

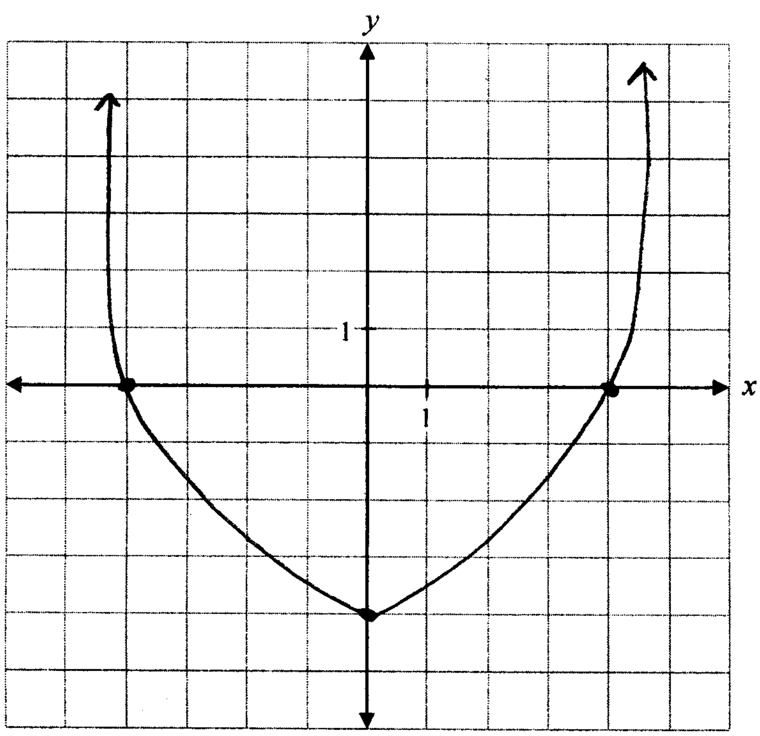
$$f(g(x)) = (x+1)^2 - 2(x+1) - 3$$
$$= (x+1)(x+1) - 2x - 2 - 3$$
$$= x^2 + x + x + 1 - 2x - 2 - 3$$
$$= x^2 + \cancel{2x} + 1 - \cancel{2x} - 2 - 3$$
$$= x^2 - 4$$
$$= (x+4)(x-4)$$
$$\therefore x = -4, 4$$

$$y\text{-int} \to (0+1)^2 - 2(0+1) - 3$$
$$= (1)^2 - 2(1) - 3$$
$$= 1 - 2 - 3$$
$$= -1 - 3$$
$$= -4$$

½ out of 1

award full marks
– ½ mark for arithmetic error in line 6

b)



1 out of 1

+ 1 mark for consistent graph

Exemplar 3

a)

$y = f(x+1)$

$f(x+1) = (x+1)^2 - 2(x+1) - 3$

$= (x+1)(x+1) - 2x - 2 - 3$

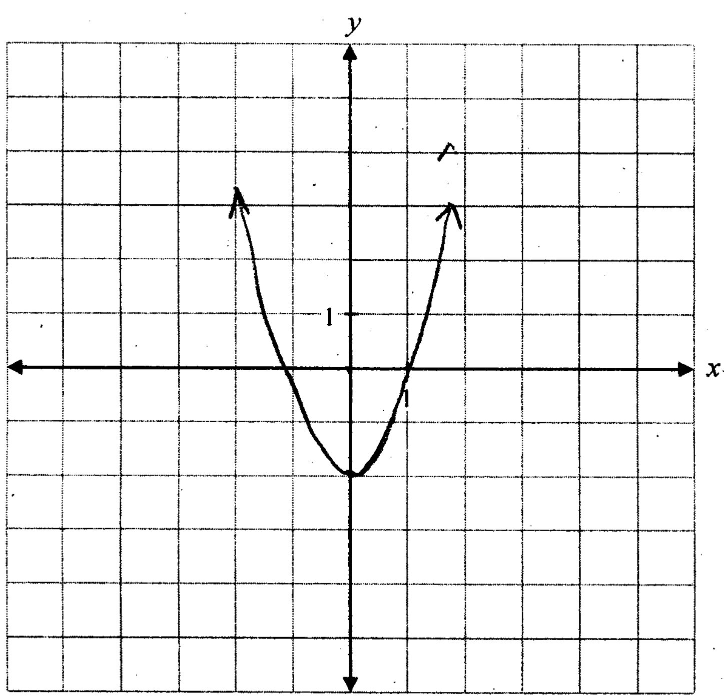
$= x^2 + \cancel{2x} + 1 - \cancel{2x} - 2 - 3$

$f(g(x)) = x^2 - 2$

½ out of 1

+ 1 mark for composition
– ½ mark for arithmetic error in line 5

b)



½ out of 1

+ ½ mark for *y*-intercept

Exemplar 4

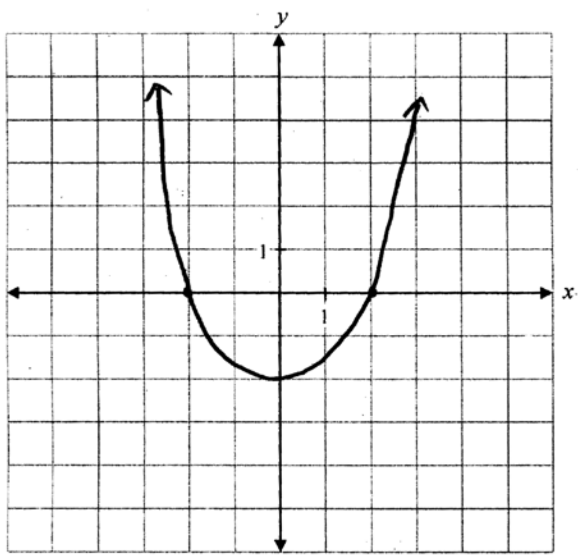
a)

$$f(x) = (x+1)^2 - 2(x+1) - 3$$
$$x^2 + 1 - 2x - 2 - 3$$
$$x^2 - 2x - 4$$
$$(x+2)(x-2)$$
$$x = -2 \quad x = 2$$

½ out of 1

+ 1 mark for composition
– ½ mark for arithmetic error in line 2
E2 (changing an equation to an expression in line 2)

b)



½ out of 1

+ ½ mark for *x*-intercepts

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Question 38 T2

Is the point $\left(\frac{3}{4}, -\frac{\sqrt{3}}{4}\right)$ on the unit circle?

Justify your answer.

Solution

$$x^2 + y^2 = 1$$

$$\text{Left-hand side } = \left(\frac{3}{4}\right)^2 + \left(-\frac{\sqrt{3}}{4}\right)^2$$

½ mark for substitution

$$= \frac{9}{16} + \frac{3}{16}$$

$$= \frac{12}{16}$$

$$\frac{12}{16} \neq 1 \therefore \text{ not on unit circle}$$

½ mark for justification

1 mark

Exemplar 1

Yes because each value is below 1

0 out of 1

Exemplar 2

The point is on the unit circle.

It would be on the unit circle because the coordinates are both less than one.

0 out of 1

Question 39 T3

Explain why the equation $\sec\theta = \frac{1}{4}$ has no solution.

Solution

The value of $\sec\theta$ cannot be between -1 and 1. **1 mark**

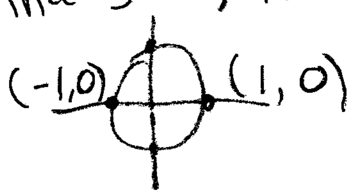
or

$\cos\theta$ cannot be greater than 1.

Exemplar 1

cos = 4

cos has a max of 1, and a min of −1 therefore you can't find sec, the reciprocal of cos.



1 out of 1

E3 (variable omitted in an equation or identity)

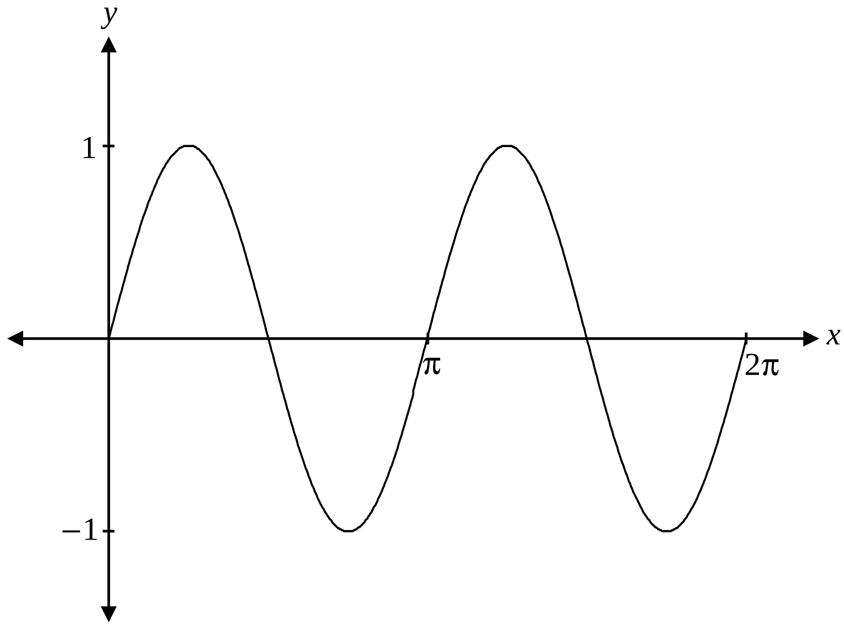
Exemplar 2

Secθ cannot be less than 1

1 out of 1

The graph of $y = \sin 2x$ is sketched below.

Explain how to use this graph to solve the equation $\sin 2x = \frac{1}{2}$ over the interval $[0, 2\pi]$.



Solution

Draw the line $y = \frac{1}{2}$. The solution will be the *x*-values where the two graphs intersect.

1 mark

Exemplar 1

you find where $y = \frac{1}{2}$ and you can figure out the intervals.

½ out of 1

award full marks
– ½ mark for terminology error in explanation

Exemplar 2

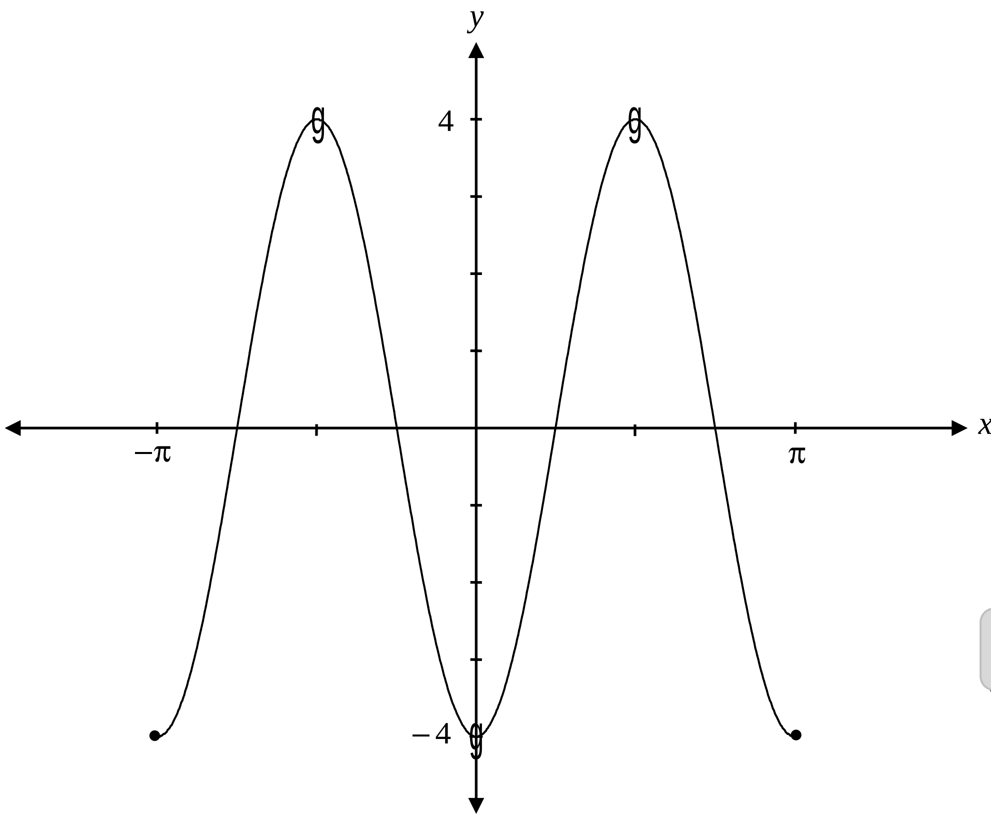
shift the graph downwards by $\frac{1}{2}$ and find the new x-intercepts

1 out of 1

Question 41 T4

Sketch the graph of $y = -4\cos(2x)$ over the interval $[-\pi, \pi]$.

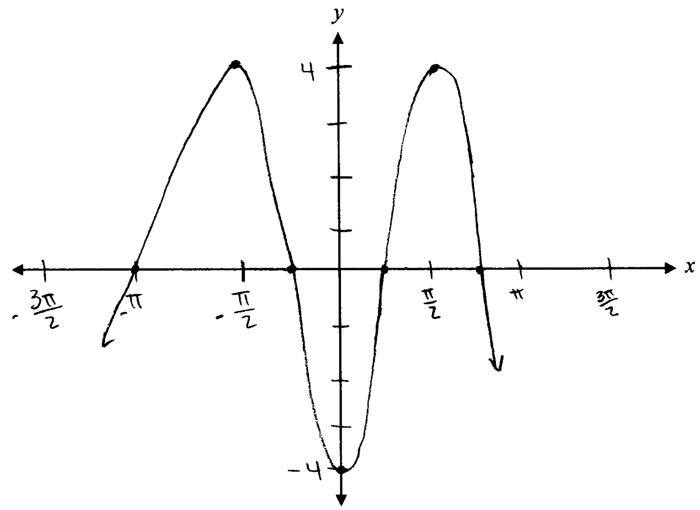
Solution



1 mark for vertical reflection
1 mark for range
1 mark for period

3 marks

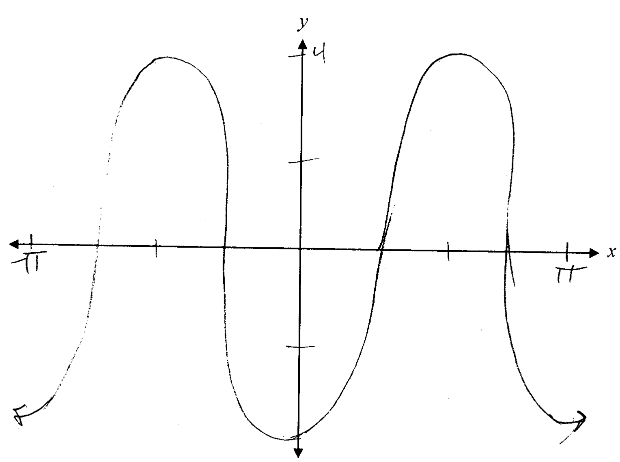
Exemplar 1



2½ out of 3

award full marks
– ½ mark for incorrect shape of graph
E9 (incorrect or missing endpoints)

Exemplar 2



2½ out of 3

award full marks
– ½ mark for incorrect shape of graph
E9 (incorrect or missing endpoints)

Write the equation for $f(x)$ that satisfies all of the following conditions:

- $f(x)$ is a polynomial function of degree 4
- $f(x)$ has a zero at 2 with a multiplicity of 3
- $f(x)$ has a zero at -5
- $f(x)$ has a y-intercept of 80

Solution

$$f(x) = a(x-2)^3(x+5)$$

$$80 = a(0-2)^3(0+5)$$

$$80 = a(-8)(5)$$

$$80 = -40a$$

$$a = -2$$

$$\therefore\ f(x) = -2(x-2)^3(x+5)$$

½ mark for the factors $(x-2)$ and $(x+5)$
½ mark for multiplicity of 3
½ mark for substitution/negative value of *a*
½ mark for value of 2 for *a*

2 marks

Exemplar 1

$$f(x) = (x-2)^3 (x+5) + 80$$

1 out of 2

+ ½ mark for the factors $(x-2)$ and $(x+5)$

+ ½ mark for multiplicity of 3

Question 43 T6

Find the exact value of $\sin\left(\frac{19\pi}{12}\right)$.

Solution

$$\sin\left(\frac{10\pi}{12}+\frac{9\pi}{12}\right)=\sin\left(\frac{5\pi}{6}+\frac{3\pi}{4}\right)$$

1 mark for combination

$$\sin\frac{19\pi}{12}=\sin\frac{5\pi}{6}\cos\frac{3\pi}{4}+\cos\frac{5\pi}{6}\sin\frac{3\pi}{4}$$

$$=\left(\frac{1}{2}\right)\left(-\frac{\sqrt{2}}{2}\right)+\left(-\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

2 marks (½ mark for each exact value)

3 marks

$$=-\frac{\sqrt{2}}{4}-\frac{\sqrt{6}}{4}$$

$$=\frac{-\sqrt{2}-\sqrt{6}}{4}$$

Note(s):

§ Other combinations are possible.

Exemplar 1

$$\sin\left(\frac{10\pi}{12}+\frac{9\pi}{12}\right) = \sin\left(\frac{5\pi}{6}+\frac{3\pi}{4}\right)$$

$$\sin\frac{19\pi}{12} = \sin\frac{5\pi}{6}\cos\frac{3\pi}{4}+\cos\frac{5\pi}{6}\sin\frac{3\pi}{4}$$

$$= \left(\frac{1}{2}\right)\left(\frac{\sqrt{2}}{2}\right) + \left(\frac{\sqrt{3}}{2}\right)\left(\frac{\sqrt{2}}{2}\right)$$

$$= \frac{\sqrt{2}+\sqrt{6}}{4}$$

2 out of 3

+ 1 mark for combination

+ ½ mark for $\sin\frac{5\pi}{6}$

+ ½ mark for $\sin\frac{3\pi}{4}$

Exemplar 2

$$\sin\left(\frac{19\pi}{12}\right)$$

$$= \sin\left(\frac{5}{6}\pi + \frac{3}{4}\pi\right)$$

$$= \sin\left(\frac{1}{2}\right)\cos\left(\frac{\sqrt{2}}{2}\right) + \sin\left(\frac{\sqrt{2}}{2}\right)\cos\left(-\frac{\sqrt{3}}{2}\right)$$

$$= \frac{\sqrt{2}}{4} - \frac{\sqrt{6}}{4}$$

$$= \frac{\sqrt{2} - \sqrt{6}}{4}$$

---

2 ½ out of 3

+ 1 mark for combination
+ ½ mark for $\sin\frac{5\pi}{6}$
+ ½ mark for $\cos\frac{5\pi}{6}$
+ ½ mark for $\sin\frac{3\pi}{4}$

E7 (notation error in line 3)

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Solve the following equation:

$$2\log_2(x-1) - \log_2(x-5) = \log_2(x+1)$$

Solution

Method 1

$$\log_2 \frac{(x-1)^2}{x-5} = \log_2(x+1)$$

2 marks for logarithmic rules (1 mark for power rule, 1 mark for quotient rule)

$$\frac{(x-1)^2}{x-5} = x+1$$

1 mark for equating arguments

$$x^2 - 2x + 1 = x^2 - 4x - 5$$

$$2x = -6$$

~~$x = -3$~~

½ mark for solving for x

$\therefore$ no solution

½ mark for no solution

4 marks

Method 2

$$2\log_2(x-1) = \log_2(x+1) + \log_2(x-5)$$

$$\log_2(x-1)^2 = \log_2(x+1)(x-5)$$

2 marks for logarithmic rules (1 mark for power rule, 1 mark for product rule)

$$(x-1)^2 = (x+1)(x-5)$$

1 mark for equating arguments

$$x^2 - 2x + 1 = x^2 - 4x - 5$$

$$2x = -6$$

~~$x = -3$~~

½ mark for solving for x

$\therefore$ no solution

½ mark for no solution

4 marks

Method 3

$$\log_2(x-1)^2 - \log_2(x-5) - \log_2(x+1) = 0$$

$$\log_2 \frac{(x-1)^2}{(x-5)(x+1)} = 0$$

2 marks for logarithmic rules (1 mark for power rule, 1 mark for quotient rule)

$$2^0 = \frac{(x-1)^2}{(x-5)(x+1)}$$

1 mark for exponential form

$$x^2 - 4x - 5 = x^2 - 2x + 1$$

$$-6 = 2x$$

$$\cancel{-3 = x}$$

½ mark for solving for x

$\therefore$ no solution

½ mark for no solution

4 marks

Exemplar 1

$$\log_2(x-1)^2 - \log_2(x-5) = \log_2(x+1)$$

$$\log_2\left(\frac{(x-1)^2}{x-5}\right) = \log_2(x+1)$$

$$\left(\frac{(x-1)^2}{(x-5)}\right) = \frac{(x+1)}{1}$$

$$x^2-2x+1 = x^2-4x-5$$

$$2x+6=0$$

$$2(x+3)=0$$

$$\boxed{x=-3}$$

$$(x-1)(x-1)$$
$$x^2-2x+1$$

$$(x+1)(x-5)$$
$$x^2 \underbrace{-5x+x}_{-4x} -5$$

3 ½ out of 4

+ 1 mark for power rule
+ 1 mark for quotient rule
+ 1 mark for equating arguments
+ ½ mark for solving for *x*

Exemplar 2

$$\log_2 \frac{(x-1)^2}{x-5} - \log_2 (x+1) = 0$$

$$\log_2 \frac{(x-1)(x-1)}{(x-5)(x+1)} = 0$$

$$2^0 = \frac{x^2-2x+1}{(x-5)(x+1)}$$

$$(x-5)(x+1) = x^2-2x+1$$

$$x^2+x-5x-5-x^2+2x-1=0$$

$$-2x-6=0$$

$$-2x=6$$

$$x=-3$$

3½ out of 4

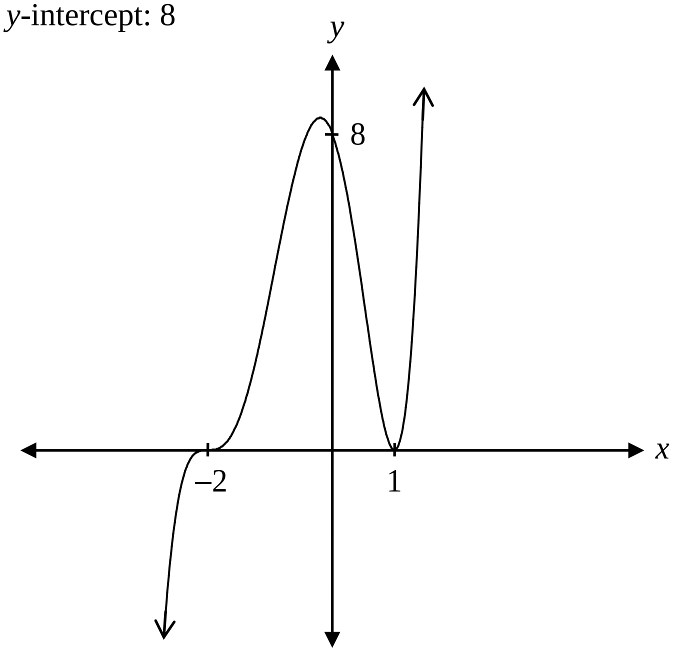
+ 1 mark for power rule
+ 1 mark for quotient rule
+ 1 mark for exponential form
+ ½ mark for solving for *x*

Sketch the graph of $f(x) = (x-1)^2(x+2)^3$.

Label the x-intercepts and the y-intercept.

Solution

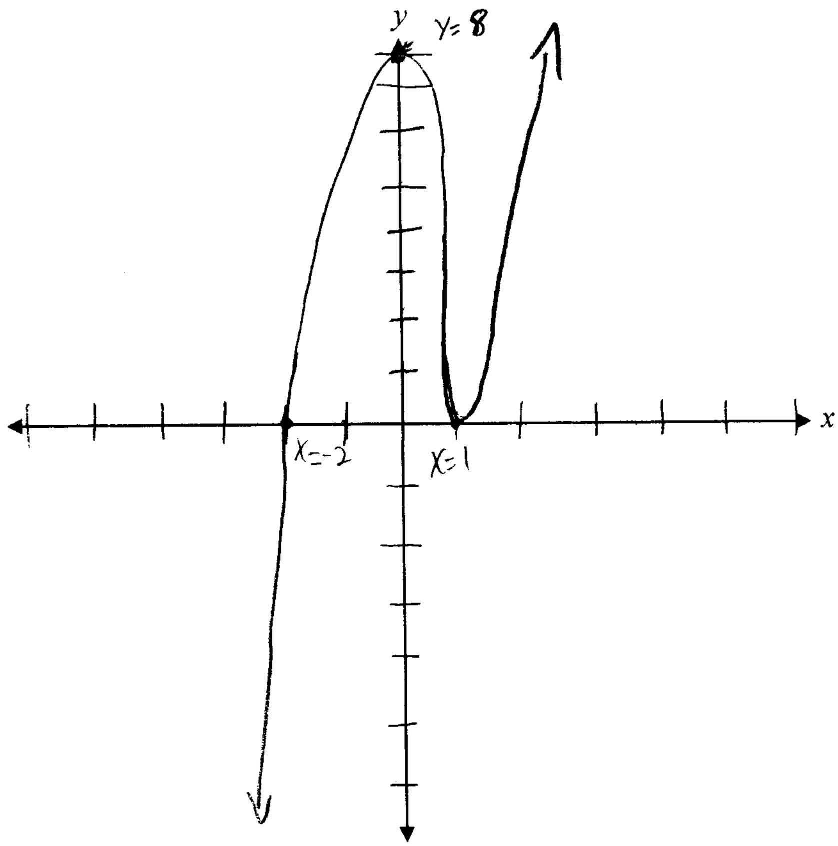
x-intercepts: $-2, 1$
y-intercept: 8



1 mark for x-intercepts
½ mark for y-intercept
1 mark for multiplicity (½ mark for degree of 2, ½ mark for degree of 3)
½ mark for end behaviour

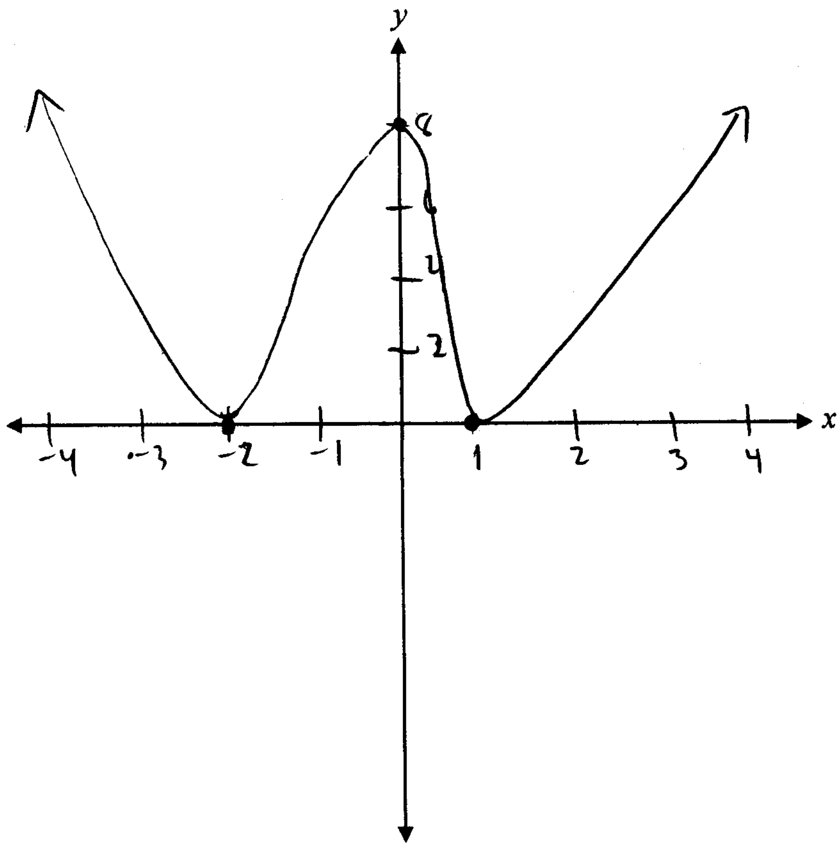
3 marks

Exemplar 1



2 ½ out of 3

+ 1 mark for *x*-intercepts
+ ½ mark for *y*-intercept
+ ½ mark for multiplicity for degree of 2
+ ½ mark for end behaviour



X-int = -2 , 1

y-int = 8

2 out of 3

+ 1 mark for *x*-intercepts
+ ½ mark for *y*-intercept
+ ½ mark for multiplicity for degree of 2

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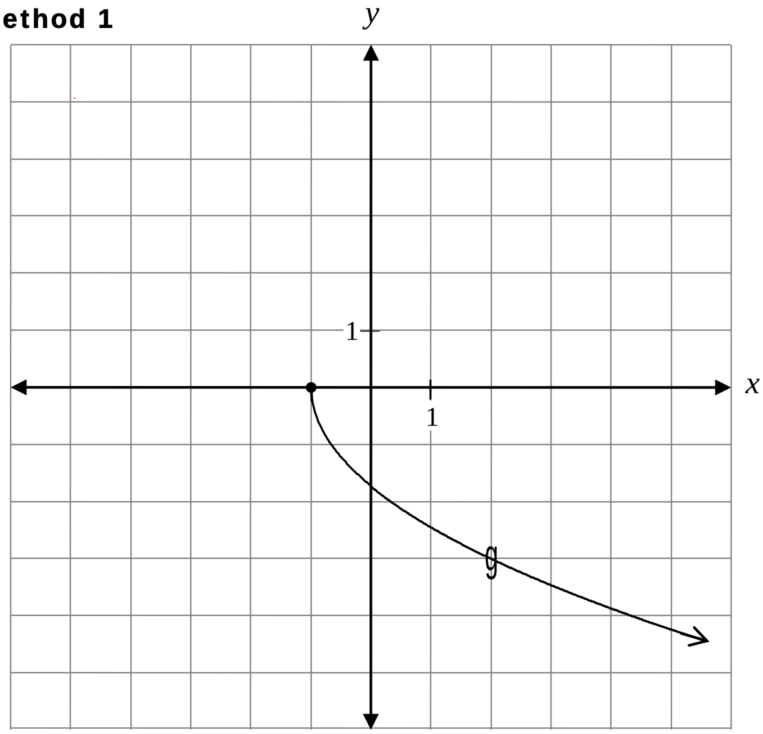
Sketch the graph of $y = -\sqrt{3(x+1)}$.

Solution

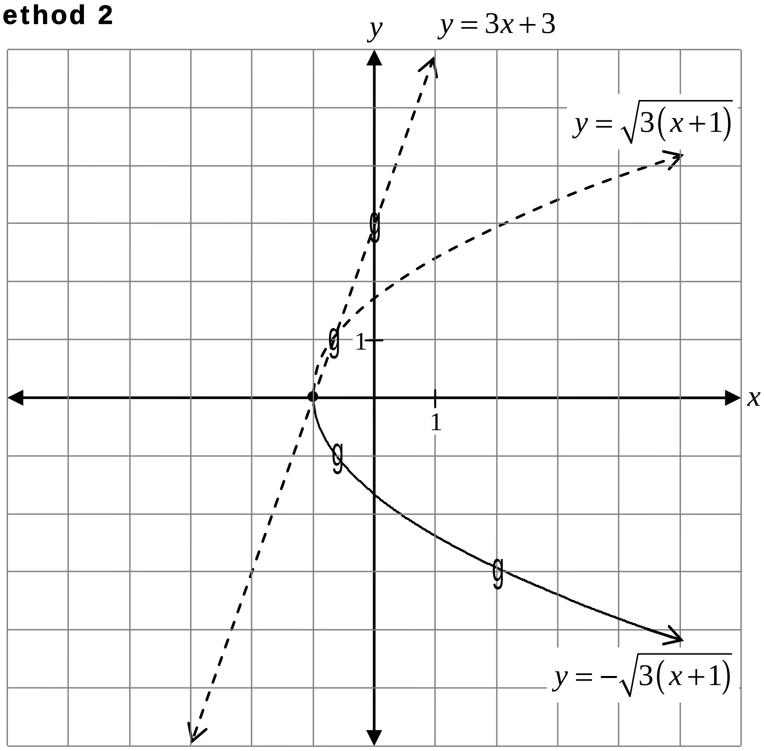
Method 1



1 mark for horizontal shift
1 mark for vertical reflection
1 mark for shape (graph of a radical function)
1 mark for horizontal compression

4 marks

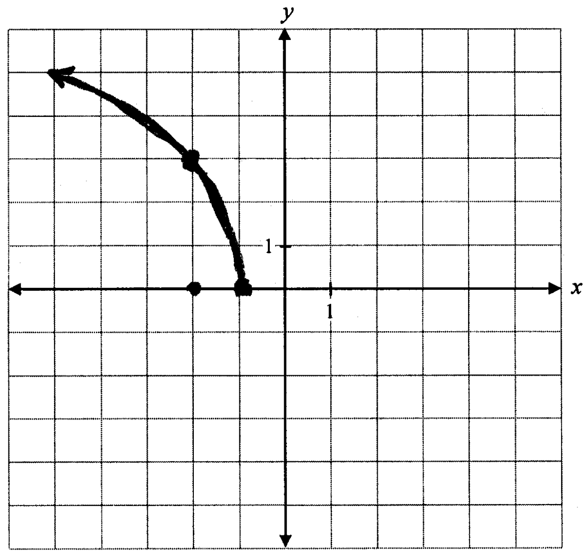
Method 2



1 mark for invariant point where $y = 0$ and $y = 1$ (½ mark for each point)
1 mark for domain of $y = -\sqrt{3(x+1)}$: $[-1, \infty)$
1 mark for reflection in the x-axis
½ mark for shape between invariant points
½ mark for shape to the right of the invariant points

4 marks

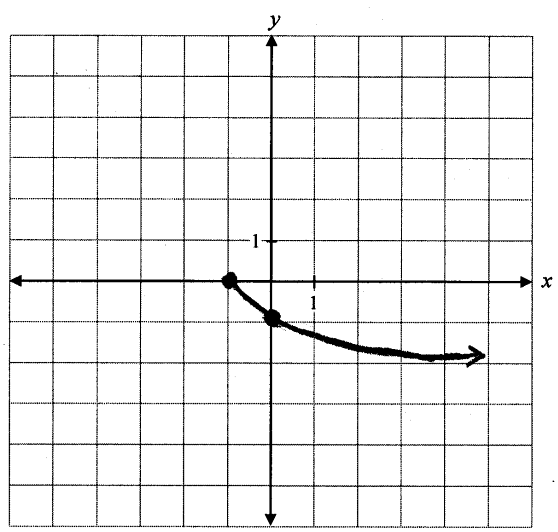
Exemplar 1



2 out of 4

+ 1 mark for horizontal shift
+ 1 mark for shape (graph of a radical function)

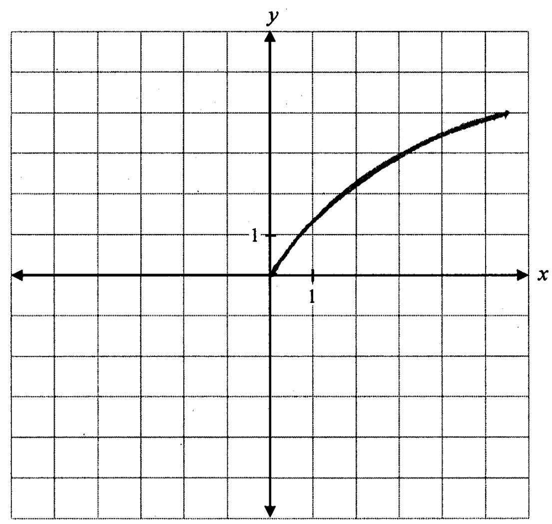
Exemplar 2



3 out of 4

+ 1 mark for horizontal shift
+ 1 mark for shape (graph of a radical function)
+ 1 mark for vertical reflection

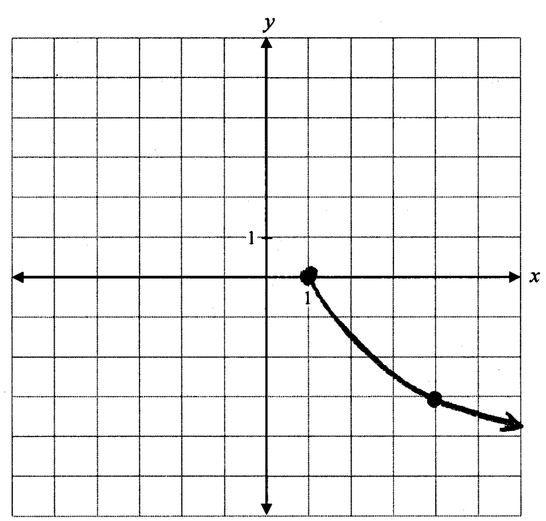
Exemplar 3



1 out of 4

+ 1 mark for shape (graph of a radical function)
E9 (missing arrowheads)

Exemplar 4



3 out of 4

+ 1 mark for vertical reflection
+ 1 mark for shape (graph of a radical function)
+ 1 mark for horizontal compression

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Solve:

$${}_{n-1}P_2 = 42$$

Solution

$\dfrac{(n-1)!}{(n-1-2)!} = 42$	½ mark for substitution
$\dfrac{(n-1)(n-2)(n-3)!}{(n-3)!} = 42$	1 mark for factorial expansion
$(n-1)(n-2) = 42$	½ mark for simplification of factorials
$n^2 - 3n + 2 = 42$	
$n^2 - 3n - 40 = 0$	
$(n-8)(n+5) = 0$	
$n = 8$ ~~$n = -5$~~	½ mark for solving for *n* ½ mark for rejecting extraneous root

3 marks

Exemplar 1

$$\frac{(n-1)!}{(n-2)!} = 42$$

$$\frac{(n-1)\cancel{(n-2)!}}{\cancel{(n-2)!}} = 42$$

$$n-1 = 42$$

$$n = 41$$

2 out of 3

+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials
+ ½ mark for solving for *n*

Exemplar 2

$$42 = \frac{n!}{(n-2)!} = \frac{(n)(n-1)\cancel{(n-2)}\ldots}{\cancel{(n-2)(n-3)(n-4)\ldots}}$$

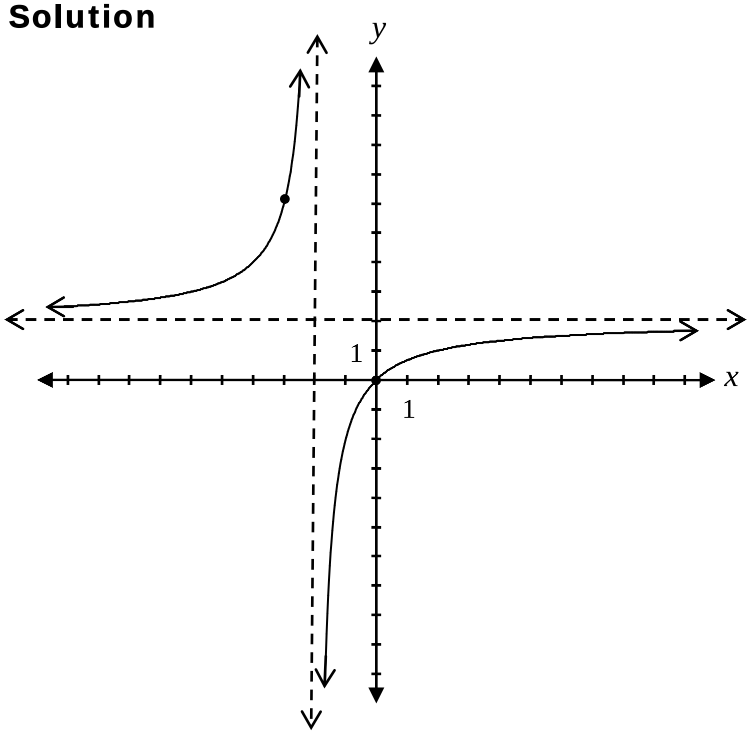
$$42 = (n)(n-1)$$

1½ out of 3

+ 1 mark for factorial expansion
+ ½ mark for simplification of factorials
E7 (notation errors in line 1)

Sketch the graph of $y = \dfrac{2x}{x+2}$.

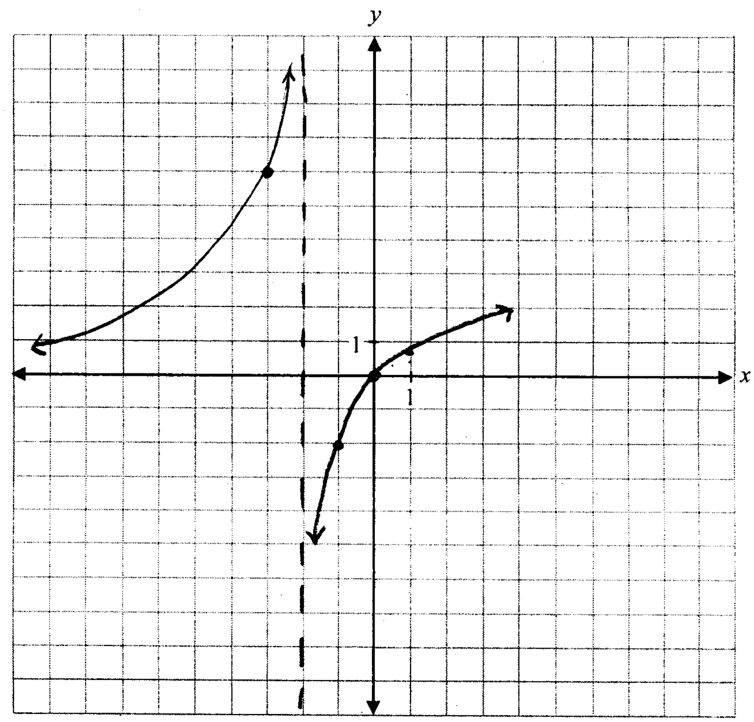
Solution



1 mark for horizontal asymptote at $y = 2$
1 mark for vertical asymptote at $x = -2$
½ mark for graph left of vertical asymptote
½ mark for graph right of vertical asymptote

3 marks

Exemplar 1



$f(-1) = \frac{2(-1)}{-1+2}$

$= \frac{-2}{1}$

$= -2$

$f(1) = \frac{2(1)}{1+2}$

$= \frac{2}{3}$

$f(-3) = \frac{-6}{-1}$

vertical asymptote

$x = -2$

x-intercept:

$(x+2)0 = \frac{2x}{x+2}(x+2)$

$0 = 2x$

$x = 0$

Discontinuity:

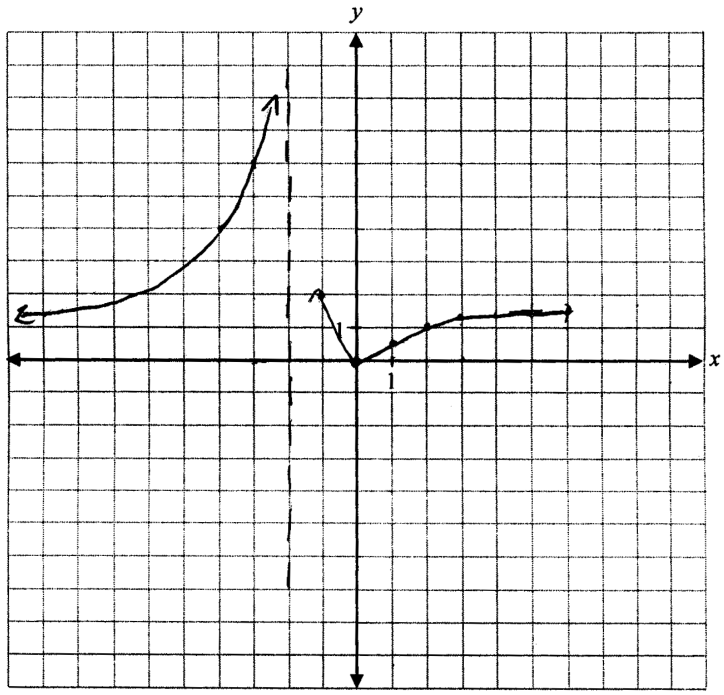
$y = \frac{2x}{x+2}$

There is none

2 out of 3

+ 1 mark for vertical asymptote at $x = -2$
+ ½ mark for graph left of vertical asymptote
+ ½ mark for graph right of vertical asymptote

Exemplar 2



x	y
−3	6
−2	imp.
−1	2
0	0
1	2/3
2	1
3	6/5
4	

1½ out of 3

+ 1 mark for vertical asymptote at $x = -2$
+ ½ mark for graph left of vertical asymptote

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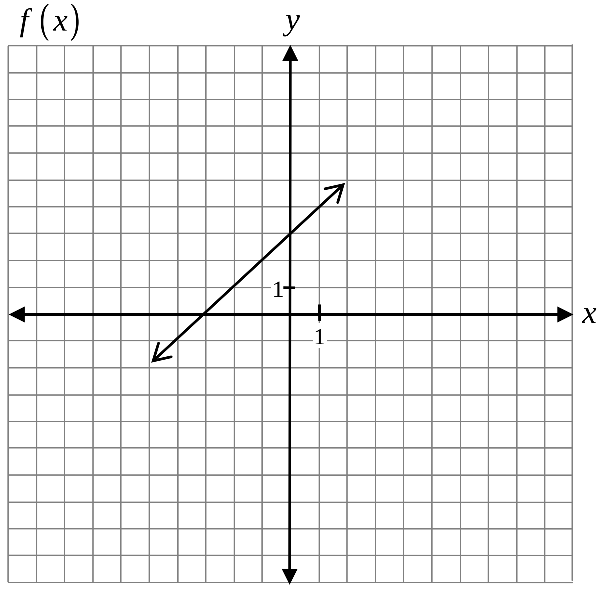
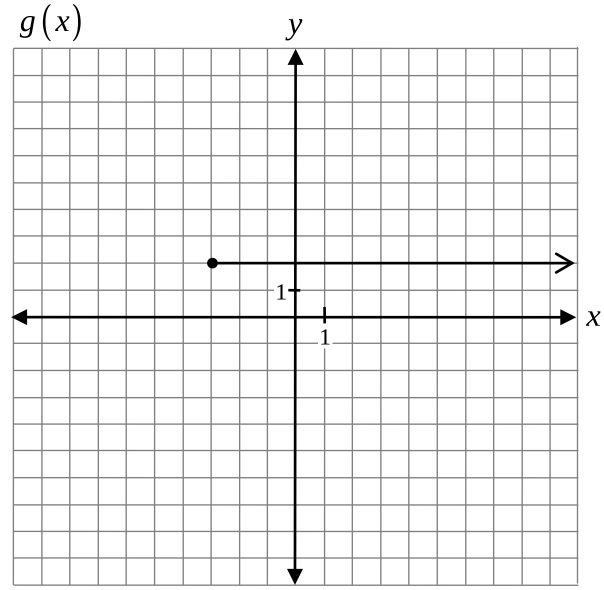
Given the graphs of $f(x)$ and $g(x)$, sketch the graph of $(f \cdot g)(x)$.

$f(x)$

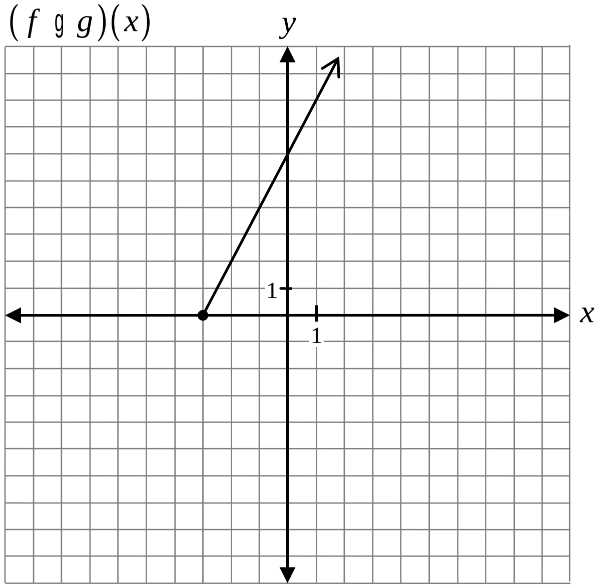


$g(x)$



Solution

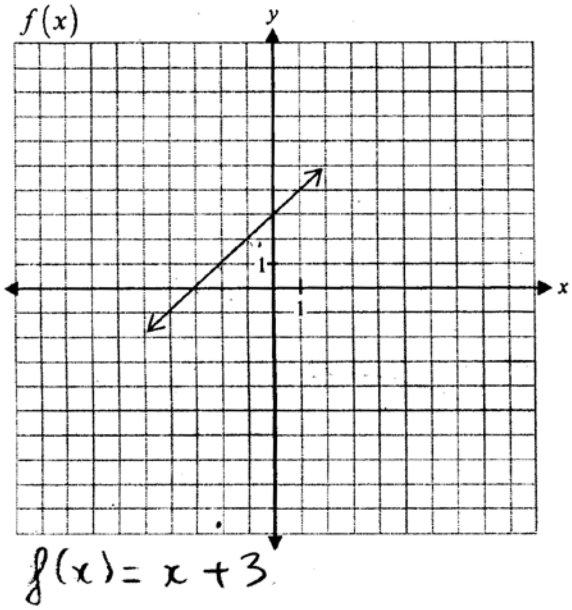
$(f \cdot g)(x)$

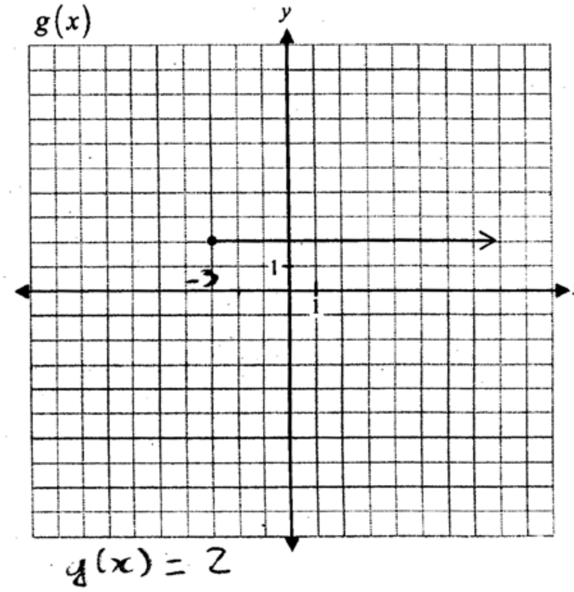


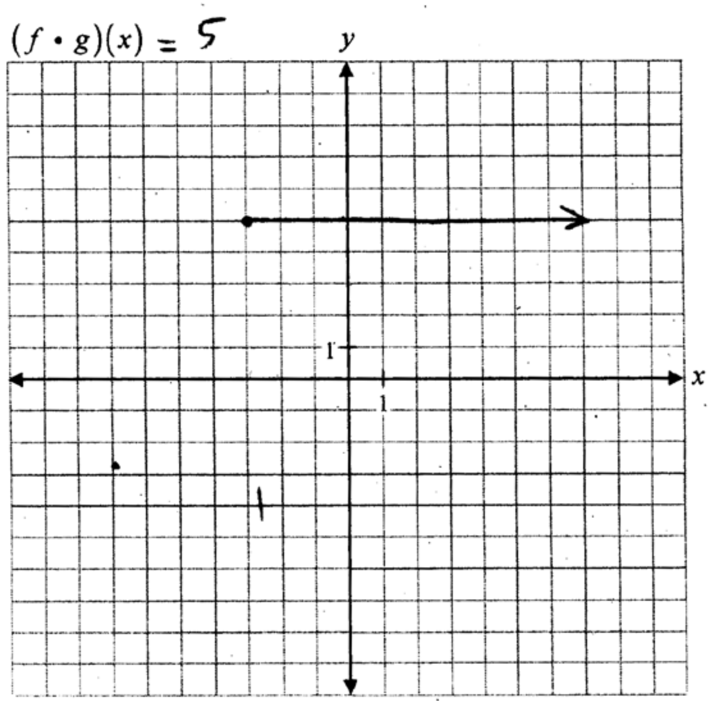
1 mark for operation of multiplication
1 mark for restricted domain

2 marks

Exemplar 1



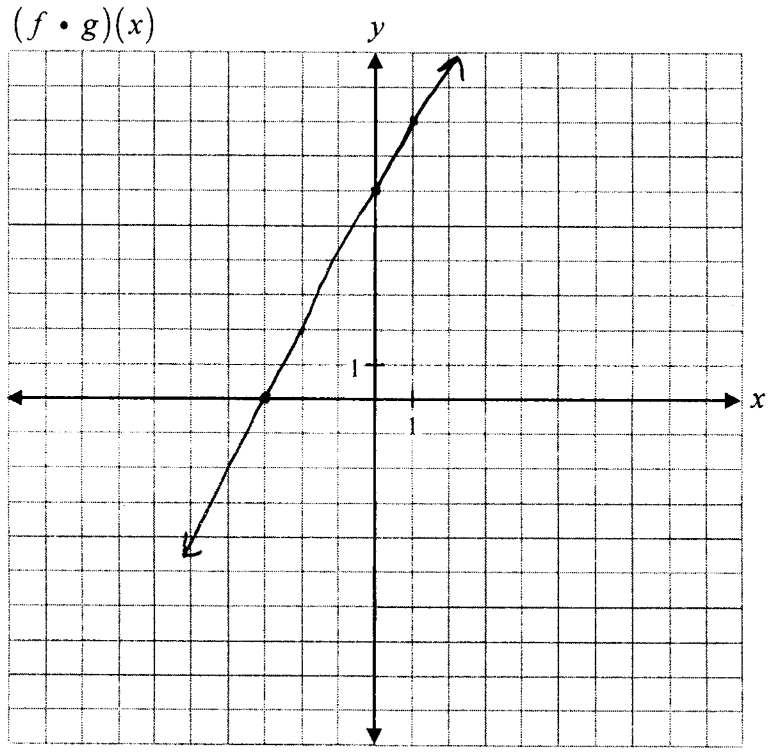




1 out of 2

+ 1 mark for restricted domain

Exemplar 2



1 out of 2

+ 1 mark for operation of multiplication

Appendices

Appendix A

MARKING GUIDELINES

Errors that are conceptually related to the learning outcomes associated with the question will result in a 1 mark deduction.

Each time a student makes one of the following errors, a ½ mark deduction will apply.

- arithmetic error
- procedural error
- terminology error in explanation
- lack of clarity in explanation
- incorrect shape of graph (only when marks are not allocated for shape)

Communication Errors

The following errors, which are not conceptually related to the learning outcomes associated with the question, may result in a ½ mark deduction and will be tracked on the *Answer/Scoring Sheet*.

E1 final answer	§ answer given as a complex fraction § final answer not stated
E2 equation/expression	§ changing an equation to an expression § equating the two sides when proving an identity
E3 variables	§ variable omitted in an equation or identity § variables introduced without being defined
E4 brackets	§ "$\sin x^2$" written instead of "$\sin^2 x$" § missing brackets but still implied
E5 units	§ missing units of measure § incorrect units of measure § answer stated in degrees instead of radians or vice versa
E6 rounding	§ rounding error § rounding too early
E7 notation/transcription	§ notation error § transcription error
E8 domain/range	§ answer included outside the given domain § bracket error made when stating domain or range § domain or range written in incorrect order
E9 graphing	§ incorrect or missing endpoints or arrowheads § scale values on axes not indicated § coordinate points labelled incorrectly
E10 asymptotes	§ asymptotes drawn as solid lines § asymptotes missing but still implied § graph crosses or curls away from asymptotes

Appendix B

IRREGULARITIES IN PROVINCIAL TESTS

A GUIDE FOR LOCAL MARKING

During the marking of provincial tests, irregularities are occasionally encountered in test booklets. The following list provides examples of irregularities for which an *Irregular Test Booklet Report* should be completed and sent to the department:

- completely different penmanship in the same test booklet
- incoherent work with correct answers
- notes from a teacher indicating how he or she has assisted a student during test administration
- student offering that he or she received assistance on a question from a teacher
- student submitting work on unauthorized paper
- evidence of cheating or plagiarism
- disturbing or offensive content
- no responses provided by the student (all “NR”) or only incorrect responses (“0”)

Student comments or responses indicating that the student may be at personal risk of being harmed or of harming others are personal safety issues. This type of student response requires an immediate and appropriate follow-up at the school level. In this case, please ensure the department is made aware that follow-up has taken place by completing an *Irregular Test Booklet Report*.

Except in the case of cheating or plagiarism where the result is a provincial test mark of 0%, it is the responsibility of the division or the school to determine how they will proceed with irregularities. Once an irregularity has been confirmed, the marker prepares an *Irregular Test Booklet Report* documenting the situation, the people contacted, and the follow-up. The original copy of this report is to be retained by the local jurisdiction and a copy is to be sent to the department along with the test materials.

Irregular Test Booklet Report

Test: ____

Date marked: ____

Booklet No.: ____

Problem(s) noted: ____

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Question(s) affected: ____

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Action taken or rationale for assigning marks: ___________________________

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Follow-up: ____

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Decision: ____

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Marker's Signature: ____

Principal's Signature: ____

For Department Use Only—After Marking Complete

Consultant: ____

Date: ____

Appendix C

Table of Questions by Unit and Learning Outcome

Unit A: Transformations of Functions		
Question	**Learning Outcome**	**Mark**
6	R1	1
9	R2, R3	2
10	R6	1
14	R5	1
15	R1	2
20	R1	1
22	R2	1
29	R5	1
34 b)	R6	1
37 a)	R1	1
37 b)	R1	1
49	R1	2
Unit B: Trigonometric Functions		
Question	**Learning Outcome**	**Mark**
1	T1	2
7	T2	2
11	T4	1
26	T4	1
30	T1	1
31	T3	2
38	T2	1
39	T3	1
41	T4	3
Unit C: Binomial Theorem		
Question	**Learning Outcome**	**Mark**
4	P1	2
5 a)	P4	3
5 b)	P4	2
13	P2,P3	1
17	P4	1
47	P2	3
Unit D: Polynomial Functions		
Question	**Learning Outcome**	**Mark**
8 a)	R11	1
8 b)	R11	1
16	R11	2
28	R12	1
42	R12	2
45	R12	3

Unit E: Trigonometric Equations and Identities		
Question	**Learning Outcome**	**Mark**
2	T5	2
19	T6	3
25	T5	1
32	T5	1
36	T5	1
40	T5	1
43	T6	3
Unit F: Exponents and Logarithms		
Question	**Learning Outcome**	**Mark**
3	R10	3
21	R8	1
27	R7	1
33	R8	2
34 a)	R9	2
35	R9	2
44	R10	4
Unit G: Radicals and Rationals		
Question	**Learning Outcome**	**Mark**
12	R13	1
18	R13	2
23	R13	1
24	R14	1
46	R13	4
48	R14	3