

Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet 1

January 2014

Manitoba Education Cataloguing in Publication Data

Grade 12 pre-calculus mathematics achievement test.
Booklet 1. January 2014 [electronic resource]

ISBN: 978-0-7711-5573-4

1. Mathematics—Examinations, questions, etc.
 2. Educational tests and measurements—Manitoba.
 3. Mathematics—Study and teaching (Secondary)—Manitoba.
 4. Calculus—Study and teaching (Secondary)—Manitoba.
 5. Mathematical ability—Testing.
1. Manitoba. Manitoba Education.
515.076

Manitoba Education
School Programs Division
Winnipeg, Manitoba, Canada

Permission is hereby given to reproduce this document for non-profit educational purposes provided the source is cited.

After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Text Book Bureau. Order online at <www.mtbb.mb.ca>.

This resource will also be available on the Manitoba Education website at <www.edu.gov.mb.ca/k12/assess/archives/index.html>.

Websites are subject to change without notice.

Disponible en français.


Available in alternate formats upon request.

Grade 12 Pre-Calculus Mathematics Achievement Test

DESCRIPTION

Time: 3 hours

	Questions	Marks	Total Marks
Booklet 1*	10 Short-Answer	17	35
	5 Long-Answer	18	
Booklet 2	10 Multiple-Choice	10	57
	10 Short-Answer	15	
	9 Long-Answer	32	
Total			92

* The first 6 questions in *Booklet 1* require a calculator. 
You will have access to your calculator for the first 45 minutes of the test.

GENERAL DIRECTIONS

- Read all instructions carefully.
- The blank pages at the back of each booklet may be used as scrap paper, but must **not** be removed from the test booklet. No marks will be given for work done on these pages.
- Note that diagrams and graphs provided in the test booklets may not be drawn to scale.
- After 45 minutes, put away your calculator. Even though you may not have finished *Booklet 1*, *Booklet 2* will be distributed at this time. You may choose to continue working on *Booklet 1* or start working on *Booklet 2*, but you will no longer have access to your calculator.

No marks will be awarded for work done on this page.

Formula Sheet

$$s = \theta r$$

$$\sin^2 \theta + \cos^2 \theta = 1$$

$$\tan^2 \theta + 1 = \sec^2 \theta$$

$$1 + \cot^2 \theta = \csc^2 \theta$$

$$\sin(\alpha - \beta) = \sin \alpha \cos \beta - \cos \alpha \sin \beta$$

$$\cos(\alpha - \beta) = \cos \alpha \cos \beta + \sin \alpha \sin \beta$$

$$\tan(\alpha - \beta) = \frac{\tan \alpha - \tan \beta}{1 + \tan \alpha \tan \beta}$$

$$\sin(\alpha + \beta) = \sin \alpha \cos \beta + \cos \alpha \sin \beta$$

$$\cos(\alpha + \beta) = \cos \alpha \cos \beta - \sin \alpha \sin \beta$$

$$\tan(\alpha + \beta) = \frac{\tan \alpha + \tan \beta}{1 - \tan \alpha \tan \beta}$$

$$\sin 2\alpha = 2 \sin \alpha \cos \alpha$$

$$\cos 2\alpha = \cos^2 \alpha - \sin^2 \alpha$$

$$\cos 2\alpha = 1 - 2 \sin^2 \alpha$$

$$\cos 2\alpha = 2 \cos^2 \alpha - 1$$

$$\tan 2\alpha = \frac{2 \tan \alpha}{1 - \tan^2 \alpha}$$

$$\log_a (MN) = \log_a M + \log_a N$$

$$\log_a \left(\frac{M}{N} \right) = \log_a M - \log_a N$$

$$\log_a (M^n) = n \log_a M$$

$$P(n, r) \text{ or } {}_n P_r = \frac{n!}{(n-r)!}$$

$$C(n, r) \text{ or } {}_n C_r = \frac{n!}{r!(n-r)!}$$

$$t_{k+1} = {}_n C_k a^{n-k} b^k$$

Terminology Sheet

Some questions may contain directing words such as *explain*, *identify*, and *justify*. These words are explained below.

Evaluate: Find the numerical value.

Explain: Use words to provide the cause or reason for the response, or to render the response more clear and understandable.

Sketch the graph: Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

Identify/Indicate: Recognize and select the answer by stating or circling it.

Justify: Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

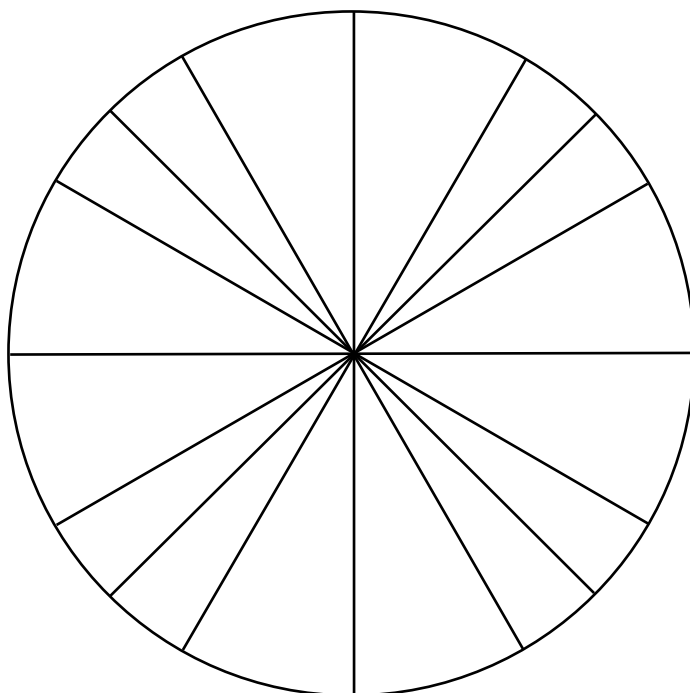
Solve: Give a solution for a problem or determine the value(s) of a variable.

Verify: Establish the truth of a statement by substitution or comparison.

Scrap Paper

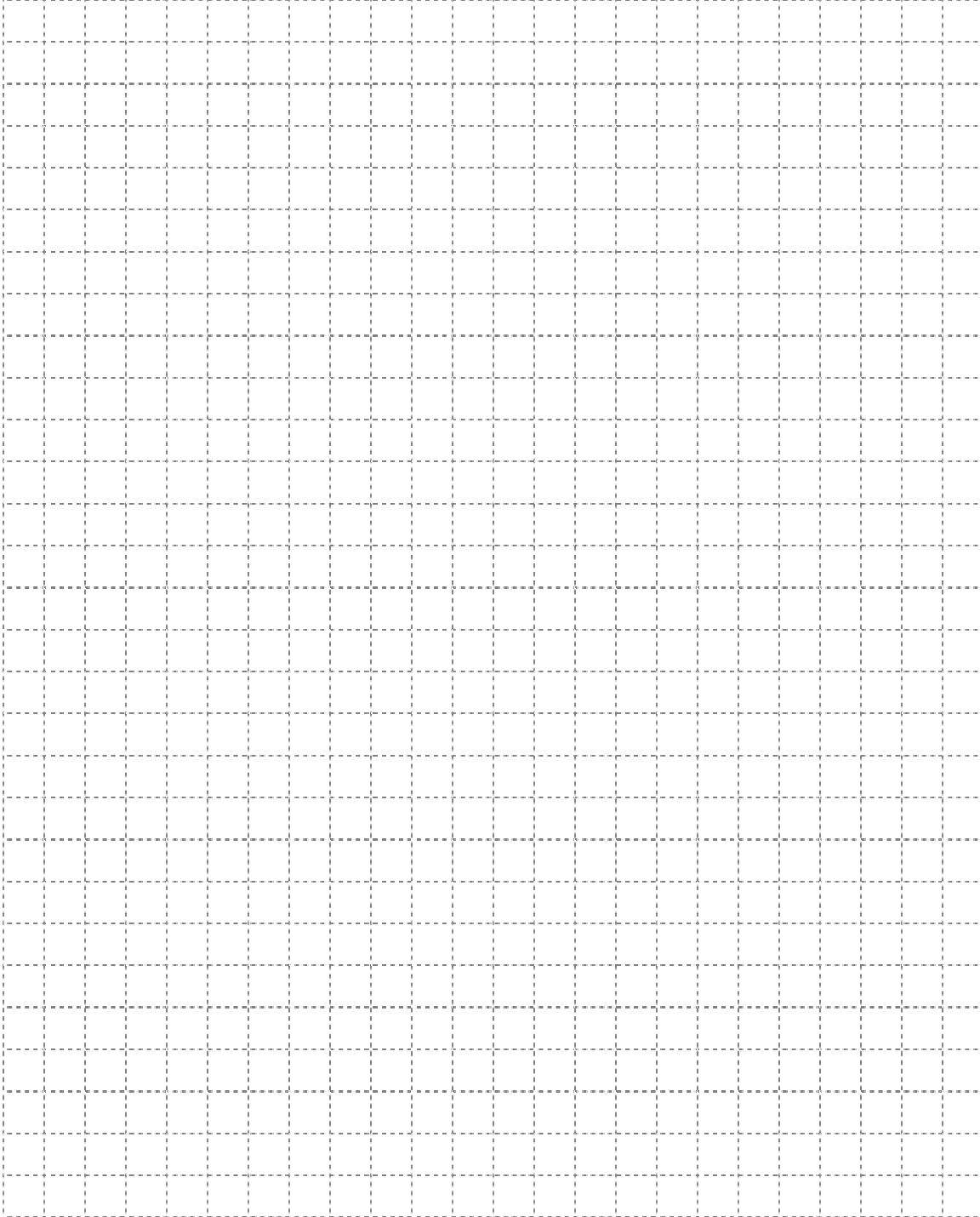
No marks will be awarded for work done on this page.

Unit Circle (can be used if needed)




Scrap Paper


No marks will be awarded for work done on this page.



Instructions

- There are 15 questions for a total of 35 marks.
- Calculators (scientific or graphing) are allowed for the first 45 minutes of the test.
- A calculator icon  appears next to the questions that require a calculator.
- Write each solution in the space provided.
- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.
- Graphing calculator solutions must include an explanation of how your final answer is obtained.
- Your solutions should be neat, organized, and clear.
- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.
- Express your answers as exact values or correct to 3 decimal places unless instructed otherwise.


No marks will be awarded for work done on this page.

Question 1 

2 marks

101

Find the coterminal angle to $\frac{27\pi}{5}$ over the interval $[-360^\circ, 0^\circ)$.

Question 2 

3 marks

102

Solve the following equation over the interval $0 \leq \theta < 2\pi$.

$$(\tan \theta - 3)(\tan \theta + 1) = 0$$

Question 3

2 marks

103

An earthquake in Vancouver had a magnitude of 6.3 on the Richter scale. An earthquake in Japan had a magnitude of 8.9 on the Richter scale.

How many times more intense was the Japan earthquake than the Vancouver earthquake?

You may use the formula below:


$$M = \log\left(\frac{A}{A_0}\right)$$

where M is the magnitude of the earthquake on the Richter scale

A is the intensity of the earthquake

A_0 is the intensity of a standard earthquake


Express your answer as a whole number.

Question 4 

2 marks

104


Find and simplify the last term in the expansion of $(2y - 3x)^7$.

Question 5 

3 marks

105

Given $\log_a 9 = 1.129$ and $\log_a 4 = 0.712$, find the value of $\log_a 12$.

Question 6 

2 marks

106

How many different ways can 4 girls and 4 boys be arranged in a row if the girls and the boys must alternate?

Note: A calculator is not required for the remaining test questions.

Question 7

4 marks

107

Solve the following equation over the interval $[0, 2\pi]$.

$$2 \cos 2\theta - 1 = 0$$

Question 8

1 mark

108

Alex incorrectly explains to Rashid that the graph of $y = 2f(x) + 5$ means you first move the graph of $y = f(x)$ up 5 units and then multiply the y values by 2.

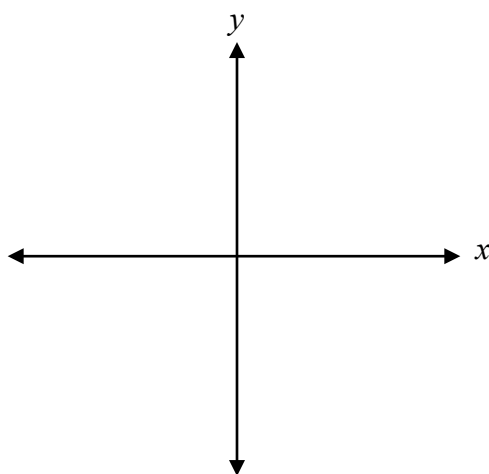
Explain to Rashid the correct way to transform the graph.

Question 9

1 mark

109

Sketch the angle of 5 radians in standard position.

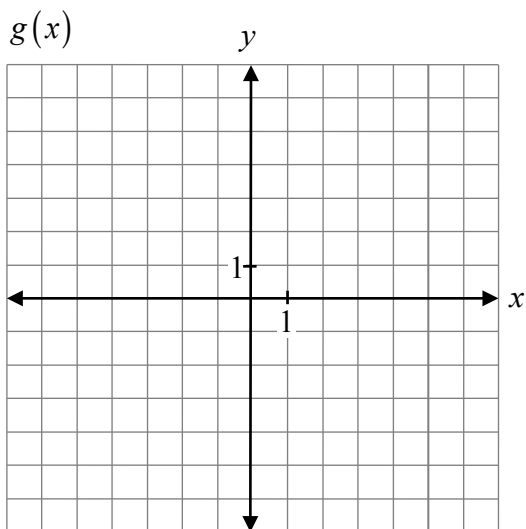
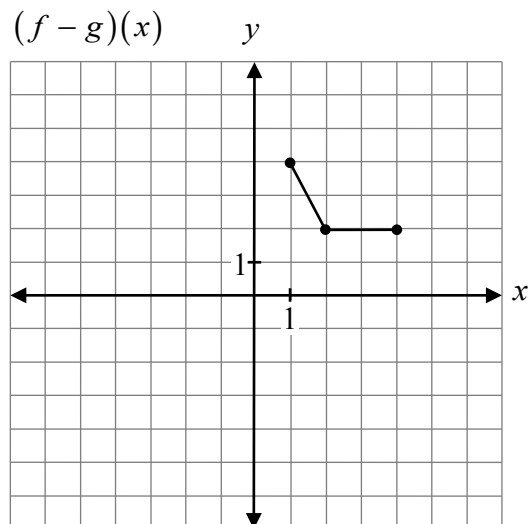
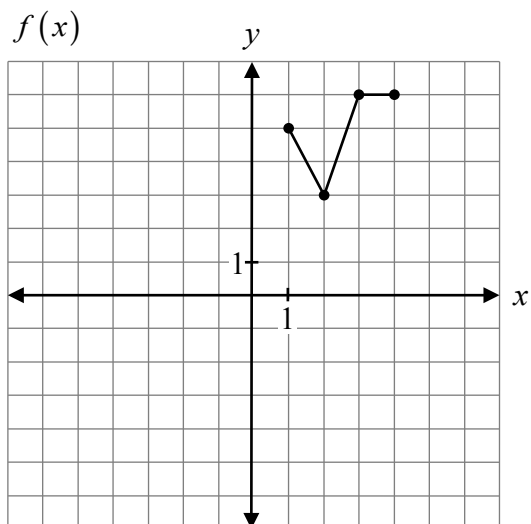


Question 10

2 marks

110

Given the graphs of $f(x)$ and $(f - g)(x)$, sketch the graph of $g(x)$.



Question 11

2 marks

111

A particular math class has a large number of students. From this class, you are to create a committee of 4 students that has at least 1 girl.

Without actually solving the problem, explain the strategy you would use to find the total number of ways to select this committee.

Question 12

a) 2 marks b) 2 marks

112
113

a) Prove the identity below for all permissible values of θ .

$$\frac{1 + 2 \cos^2 \theta}{\cos^2 \theta} = \tan^2 \theta + 3$$

Left-Hand Side	Right-Hand Side

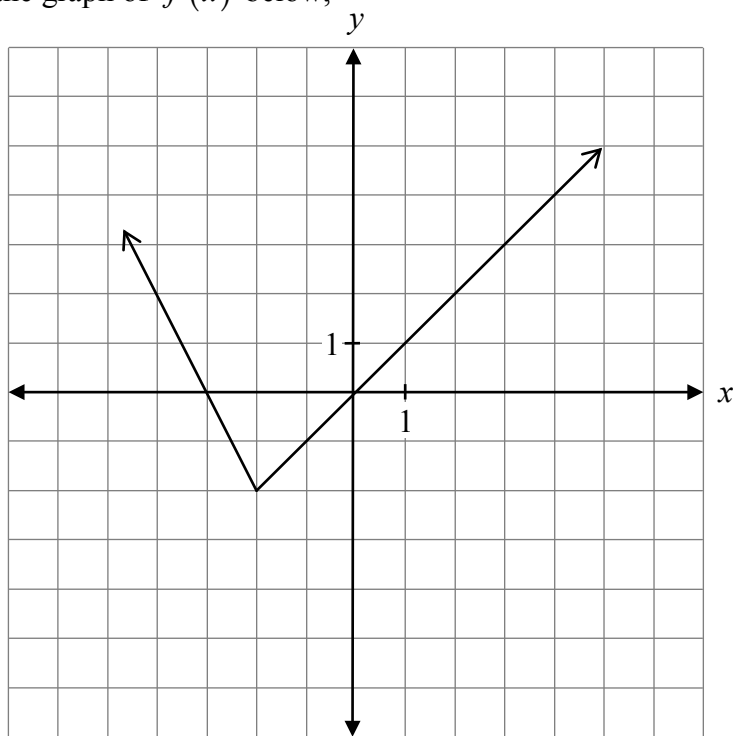
b) Determine all the non-permissible values for θ .

Question 13

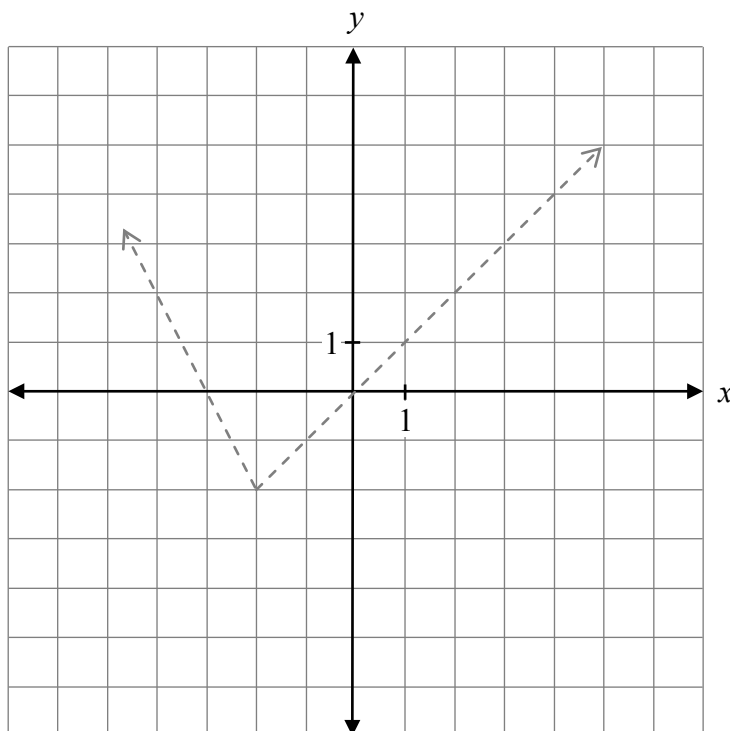
2 marks

114

Given the graph of $f(x)$ below,



sketch the graph of $g(x) = f(x - 2) + 3$.



The graph of $f(x)$ has already been drawn for your reference.

No marks will be awarded for the graph of $f(x)$.

Question 14

a) 3 marks b) 1 mark

115
116

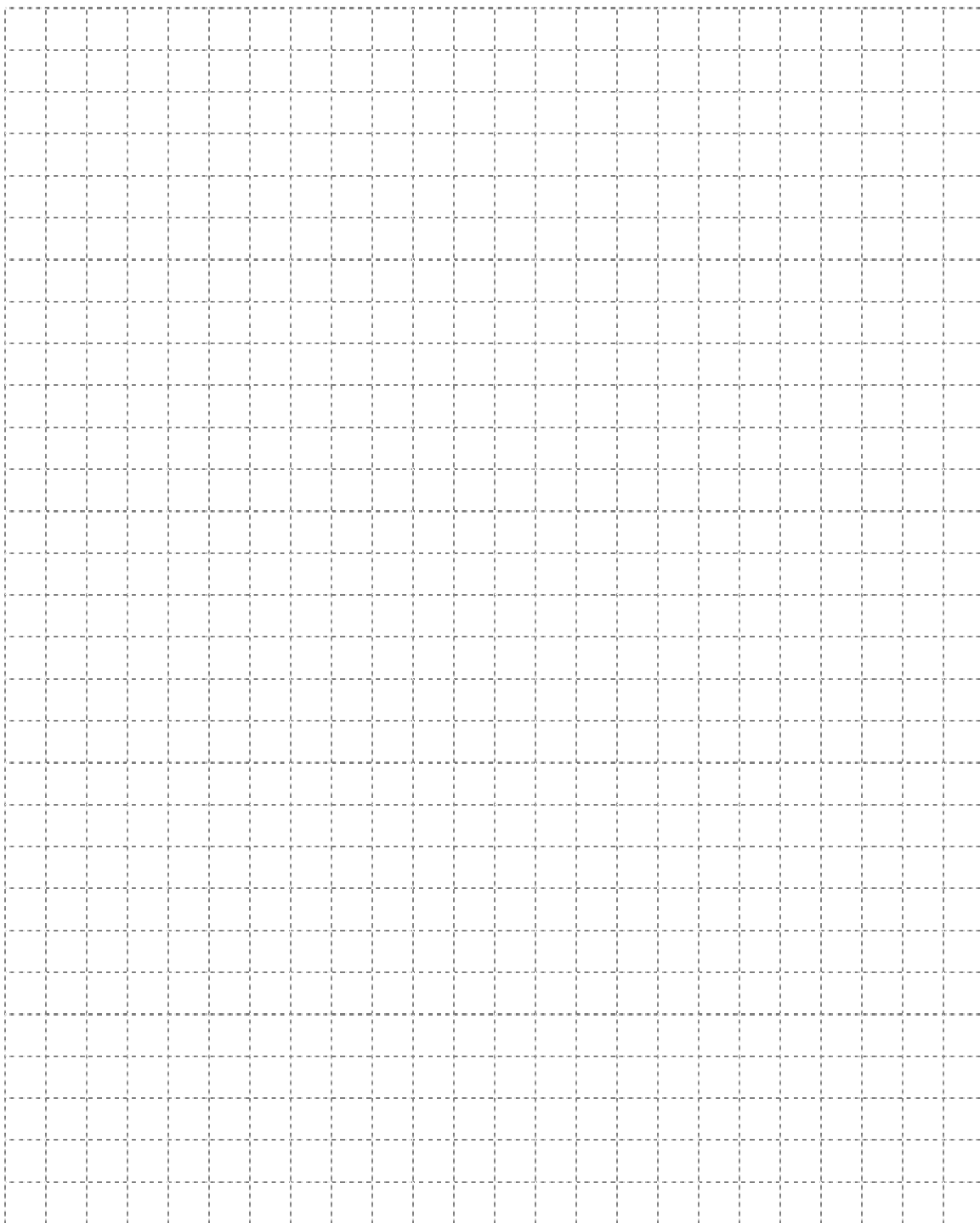
Given that $\sin \alpha = \frac{5}{13}$, where α is in Quadrant II, and $\cos \beta = \frac{2}{5}$, where β is in Quadrant IV, find the exact value of:

a) $\cos(\alpha + \beta)$

b) $\sin 2\alpha$

If $f(x) = x^3$ and $g(x) = 2x - 3$, what is the value of $\left(\frac{f}{g}\right)(-1)$?

No marks will be awarded for work done on this page.



No marks will be awarded for work done on this page.