Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet 1

June 2018

Manitoba

After the administration of this test, print copies of this resource will be available for purchase from the Manitoba Learning Resource Centre. Order online at www.manitobalrc.ca.

This resource will also be available on the Manitoba Education and Training website at www.edu.gov.mb.ca/k12/assess/archives/index.html.

Websites are subject to change without notice.

Disponible en français.

While the department is committed to making its publications as accessible as possible, some parts of this document are not fully accessible at this time.

Available in alternate formats upon request.

Grade 12 Pre-Calculus Mathematics Achievement Test

DESCRIPTION

Time: 3 hours

Numbers and Marks by Question Type

	Selected Response	Constructed Response	Marks
Booklet 1*	–	17	35
Booklet 2	9	22	54
Total	9	39	**89**

* The first 6 questions in *Booklet 1* require a calculator.
You will have access to your calculator for the first 45 minutes of the test.

Note that diagrams and graphs provided in the test booklets may not be drawn to scale.

DIRECTIONS

- Write each solution in the space provided.

- For full marks, your answers must show all pertinent diagrams, calculations, and explanations.

- Graphing calculator solutions must include an explanation of how your final answer is obtained.

- Your solutions should be neat, organized, and clear.

- Some answers are to be given as decimal values. Rounding too early in your solution may result in an inaccurate final answer for which full marks will not be given.

- Express your answers as exact values or correct to the nearest thousandth (3 decimal places) unless instructed otherwise.

Electronic communication between students through phones, email, or file sharing during the test is strictly prohibited.

No marks will be awarded for work done on this page.

Formula Sheet

$s = \theta r$

$\sin^2\theta + \cos^2\theta = 1$

$\tan^2\theta + 1 = \sec^2\theta$

$1 + \cot^2\theta = \csc^2\theta$

$\sin(\alpha - \beta) = \sin\alpha\cos\beta - \cos\alpha\sin\beta$

$\cos(\alpha - \beta) = \cos\alpha\cos\beta + \sin\alpha\sin\beta$

$\tan(\alpha - \beta) = \dfrac{\tan\alpha - \tan\beta}{1 + \tan\alpha\tan\beta}$

$\sin(\alpha + \beta) = \sin\alpha\cos\beta + \cos\alpha\sin\beta$

$\cos(\alpha + \beta) = \cos\alpha\cos\beta - \sin\alpha\sin\beta$

$\tan(\alpha + \beta) = \dfrac{\tan\alpha + \tan\beta}{1 - \tan\alpha\tan\beta}$

$\sin 2\alpha = 2\sin\alpha\cos\alpha$

$\cos 2\alpha = \cos^2\alpha - \sin^2\alpha$

$\cos 2\alpha = 1 - 2\sin^2\alpha$

$\cos 2\alpha = 2\cos^2\alpha - 1$

$\tan 2\alpha = \dfrac{2\tan\alpha}{1 - \tan^2\alpha}$

$\log_a(MN) = \log_a M + \log_a N$

$\log_a\left(\dfrac{M}{N}\right) = \log_a M - \log_a N$

$\log_a(M^n) = n\log_a M$

$P(n,r) \text{ or } {}_nP_r = \dfrac{n!}{(n-r)!}$

$C(n,r) \text{ or } {}_nC_r = \dfrac{n!}{r!(n-r)!}$

$t_{k+1} = {}_nC_k a^{n-k}b^k$

For $ax^2 + bx + c = 0$,

$x = \dfrac{-b \pm \sqrt{b^2 - 4ac}}{2a}$

Terminology Sheet

Some questions may contain directing words such as *explain*, *identify*, and *justify*. These words are defined below.
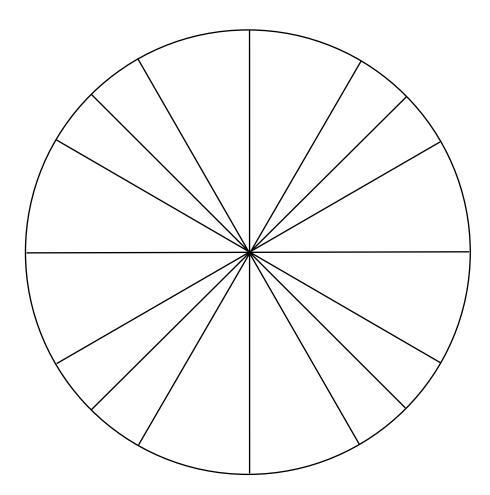
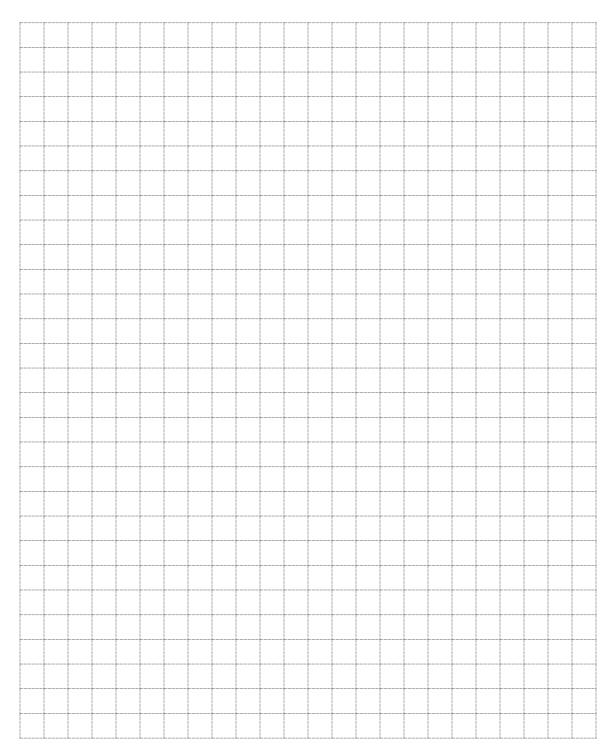**Describe:** Use words to provide the process or to report details of the response.

Determine: Use a mathematical formula, an algebraic equation, or a numerical calculation to solve a problem.

Evaluate: Find the numerical value.

Explain: Use words to provide the cause of or reason for the response, or to render the response more clear and understandable.

Identify/Indicate: Recognize and select the answer by stating or circling it.

Justify: Show reasons for or give facts that support a position by using mathematical computations, words, and/or diagrams.

Sketch the graph: Provide a detailed drawing with key features of the graph that includes a minimum of 2 coordinate points.

Solve: Give a solution for a problem or determine the value(s) of a variable.

State: Give an answer without an explanation or justification.

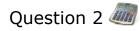**Verify:** Establish the truth of a statement by substitution or comparison.

No marks will be awarded for work done on this page.

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Pierre pushes his car into a garage. The radius of a tire on his car is 22 cm. Determine the distance travelled by his car if the tire rotated a total of 1000°.

Solve, algebraically.

$$7^{\frac{x}{2}} = 85$$

Solve, algebraically, over the interval $[0, 2\pi)$.

$$\sin x \left(\sec x + 3\right) = 0$$

Brahim invests $2500 at an annual interest rate of 6.75% compounded monthly. Determine, algebraically, how many years it will take for his investment to reach an amount of $10 500.

Use the formula:

$$A = P\left(1 + \frac{r}{n}\right)^{nt}$$

where A = the amount of the investment after t years

         P = the principal of the investment

         r = the annual interest rate (as a decimal)

         n = the number of compounding periods per year

         t = the length of the investment in years

There are 13 adults and 18 children who can be selected to go on a trip. Determine the number of ways 4 adults and 7 children can be selected if Sandra, one of the adults, must be selected.

In the binomial expansion of $\left(\dfrac{2}{x^2} - x^3 \right)^9$, determine and simplify the 6th term.

> **Note: A calculator is not required for the remaining test questions.**

Given that $f(x) = \{(-1, 0), (0, 2), (1, -3), (2, 4)\}$, evaluate $f(f(0))$.

The point $\left(-\dfrac{5}{6}, b\right)$ is on the unit circle and is in quadrant III.

Determine the exact value of b.

Given the following row of Pascal's Triangle, determine the values of the next row.
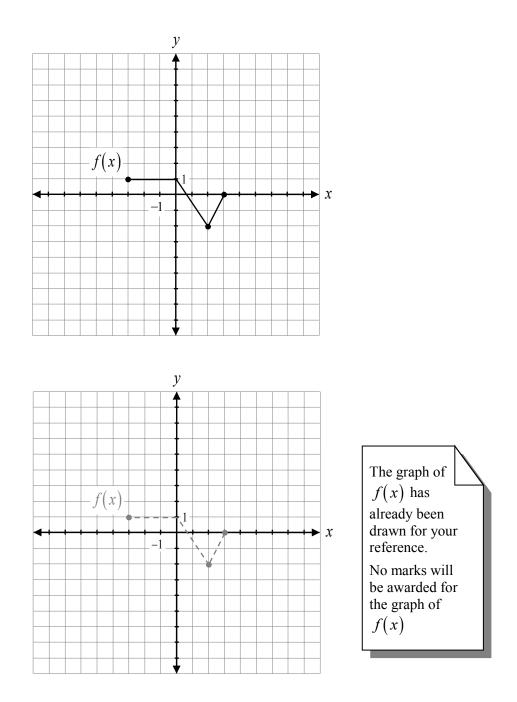
    1   6   15   20   15   6   1

The following transformations are applied to $f(x)$, resulting in a new function, $g(x)$.

- reflection over the x-axis
- vertical stretch by a factor of 3
- horizontal stretch by a factor of 4

State the equation of $g(x)$ in terms of $f(x)$.

$g(x) = $ _____

Given the graph of $f(x)$, sketch the graph of $y + 1 = 2f(x - 3)$.

$f(x)$

$f(x)$

The graph of $f(x)$ has already been drawn for your reference.

No marks will be awarded for the graph of $f(x)$

State the equation of the horizontal asymptote of $f(x) = \dfrac{2x^2 - 3x + 5}{4x^2 + 2x - 7}$.

Given that $(x + 4)$ is one of the factors of $p(x) = x^3 + 6x^2 - 32$, express $p(x)$ in completely factored form.

$p(x) =$ _____

Prove the identity for all permissible values of θ.

$$\frac{2\cos^2\theta}{1-\cot\theta} = \frac{\sin 2\theta}{\tan\theta - 1}$$

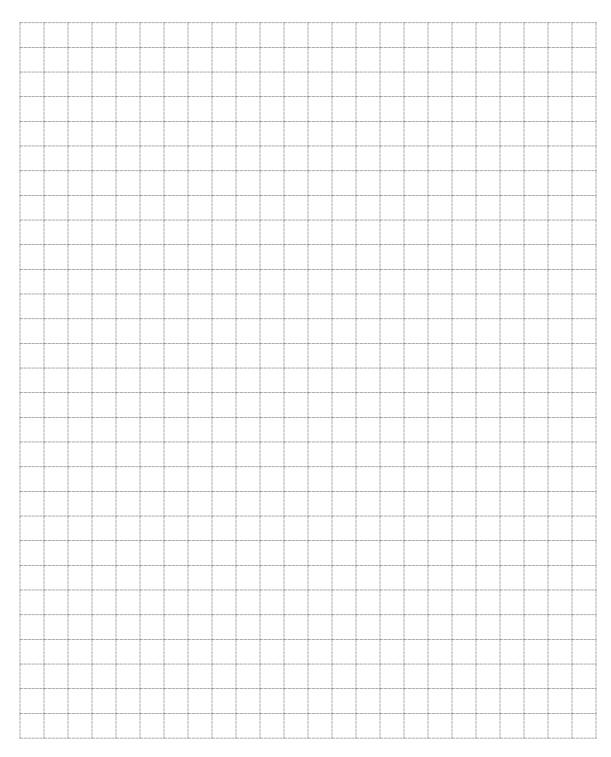| Left-Hand Side	Right-Hand Side

Restaurant A has 5 types of hamburgers, 2 types of french fries, and 10 types of drinks.
Restaurant B has 4 types of hamburgers, 5 types of french fries, and 6 types of drinks.

If a meal is made up of a hamburger, french fries, and a drink, justify which restaurant offers a greater variety of meals.

Express $\log_7(2x-5) + 2\log_7 3$ as a single logarithm.

Explain why 11! is not the total number of 11-letter arrangements that can be made from the word CELEBRATION.

No marks will be awarded for work done on this page.