Grade 12
Pre-Calculus Mathematics
Achievement Test

Booklet 2

June 2014

Manitoba

Manitoba Education and Advanced Learning Cataloguing in Publication Data

Grade 12 pre-calculus mathematics achievement test.
Booklet 2. June 2014 [electronic resource]

ISBN: 978-0-7711-5589-5

1. Mathematics—Examinations, questions, etc.
2. Educational tests and measurements—Manitoba.
3. Mathematics—Study and teaching (Secondary)—Manitoba.
4. Calculus—Study and teaching (Secondary)—Manitoba.
5. Mathematical ability—Testing.
I. Manitoba. Manitoba Education and Advanced Learning.
515.076

Manitoba Education and Advanced Learning
School Programs Division
Winnipeg, Manitoba, Canada

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Disponible en français.

Available in alternate formats upon request.

Instructions

Multiple-Choice Questions

- There are 10 questions each worth one mark.
- Calculators are **not** allowed for this part of the test.
- You may use the spaces beside each question for rough work.
- Provide only one answer per question.
- There is no penalty for guessing.
- Record your answers on the sheet provided.

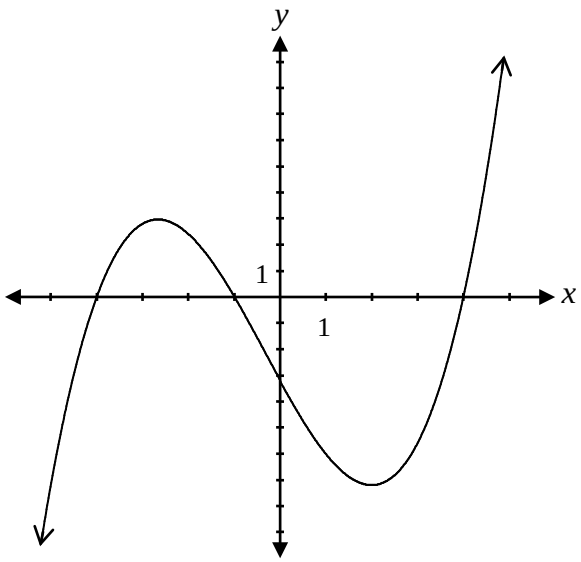
Short and Long Answer Questions

- There are 20 questions worth a total of 44 marks.
- Calculators are **not** allowed for this part of the test.
- For full marks, your answer must show all pertinent diagrams, calculations, and explanations.
- Your solutions should be neat, clear, and well organized.
- Write each solution in the space provided.

No marks will be awarded for work done on this page.

Question 20 — 1 mark

Given the graph of the function of $f(x)$ below, what is the range of $y = |f(x)|$?



a) $y \in ¡$ b) $y \geq -7$ c) $y \geq 0$ d) $-4 \leq y \leq -1$ or $y \geq 4$

Question 21 — 1 mark

Simplify the following expression:

$$\frac{1}{2}\log_a 36 - \log_a 2$$

a) $\log_a 3$ b) $\log_a 4$ c) $\log_a 9$ d) $\log_a 12$

Question 22 — 1 mark

Given $f(x) = x^2 - x + 2$, an equation that represents the graph of $f(x)$ shifted 3 units to the right is:

a) $y = (x+3)^2 - (x+3) - 3$

b) $y = (x-3)^2 - (x-3) + 2$

c) $y = (x-3)^2 - x - 2$

d) $y = x^2 - x + 2 - 3$

Question 23 — 1 mark

What is the domain of the function $y = \sqrt{-4x}$?

a) $\{x \in \mathbb{I} \mid x \geq 2\}$

b) $\{x \in \mathbb{I} \mid x \leq 2\}$

c) $\{x \in \mathbb{I} \mid x \geq 0\}$

d) $\{x \in \mathbb{I} \mid x \leq 0\}$

Question 24 — 1 mark

Which of the following is true about the two functions below?

$$f(x) = \frac{(x+2)(x-2)}{x-2} \qquad g(x) = \frac{(x-2)(x+1)}{(x+2)(x-2)}$$

a) Both have a point of discontinuity (hole) when $x = 2$.

b) Both have the same vertical asymptote.

c) Both have the same horizontal asymptote.

d) Both have the same *y*-intercept.

Question 25 — 1 mark

The general solution to the equation $\cos\theta = -\frac{1}{2}$ is:

a) $\left.\begin{array}{l}\theta = \frac{\pi}{3} + 2\pi k \\ \theta = \frac{5\pi}{3} + 2\pi k\end{array}\right\}$ where $k \in \text{I}$

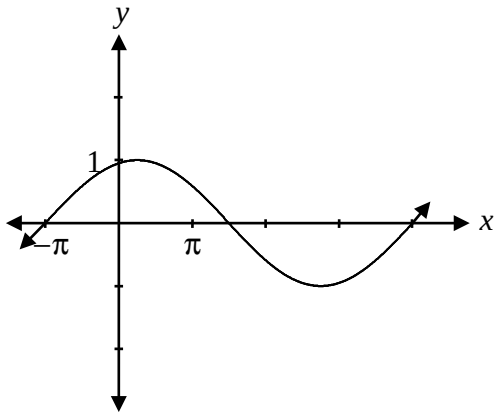
b) $\left.\begin{array}{l}\theta = \frac{\pi}{3} + \pi k \\ \theta = \frac{5\pi}{3} + \pi k\end{array}\right\}$ where $k \in \text{I}$

c) $\left.\begin{array}{l}\theta = \frac{2\pi}{3} + 2\pi k \\ \theta = \frac{4\pi}{3} + 2\pi k\end{array}\right\}$ where $k \in \text{I}$

d) $\left.\begin{array}{l}\theta = \frac{2\pi}{3} + \pi k \\ \theta = \frac{4\pi}{3} + \pi k\end{array}\right\}$ where $k \in \text{I}$

Question 26 — 1 mark

If the equation $y = \sin\left(b\left(x + \pi\right)\right)$ is represented by the following graph, what is the value of b?



a) $\frac{2}{5}$ b) $\frac{5}{2}$ c) $\frac{2\pi}{5}$ d) 5π

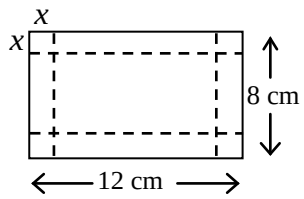
Question 27 — 1 mark

Which of the following is closest to the value of $\log_2 40 + \log_5 125$?

a) 3 b) 8 c) 10 d) 45

Question 28 — 1 mark

A sheet of paper 12 cm long and 8 cm wide is used to make a box with no lid. Equal squares of side length x cm are cut from each of the corners and the sides are folded up to make the box.



Which of the following expresses the volume of the box?

a) $V(x) = x(12 + x)(8 + x)$

b) $V(x) = x(12 - x)(8 - x)$

c) $V(x) = x(12 + 2x)(8 + 2x)$

d) $V(x) = x(12 - 2x)(8 - 2x)$

Question 29 — 1 mark

Given that the graph of $f(x)$ contains the point $(-3, 5)$, what point must be on the graph of $f(-x)$?

a) $(-3, -5)$

b) $(3, 5)$

c) $(3, -5)$

d) $(5, -3)$

Question 30 — 1 mark — 122

Determine one positive and one negative coterminal angle with the angle $\frac{5\pi}{6}$.

Question 31 — 2 marks — 123

Evaluate:

$$\left(\sin\frac{11\pi}{3}\right)\left(\sec\frac{11\pi}{6}\right)$$

Question 32 — 1 mark — 124

Given the equation $2\sin^2\theta - 3\sin\theta + 1 = 0$, verify that $\theta = \frac{\pi}{2}$ is a solution.

Question 33 — 2 marks — 125

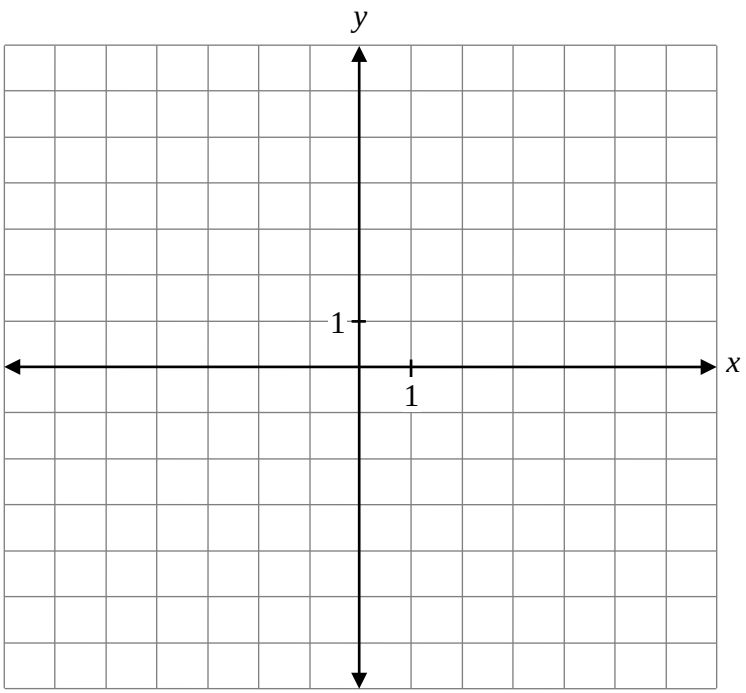
Using the laws of logarithms, expand:

$$\log_a\left(\frac{x \cdot y}{z}\right)$$

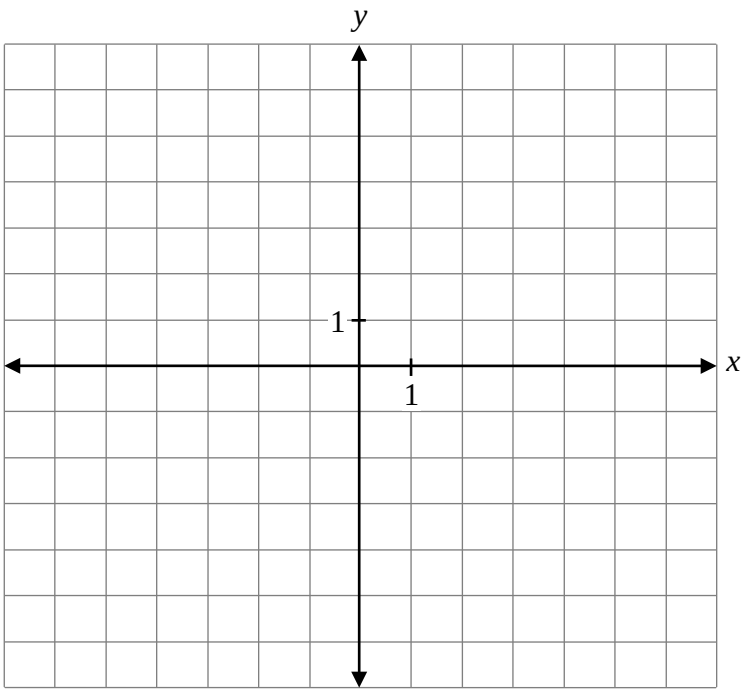
Question 34 — a) 2 marks b) 1 mark

126
127

a) Sketch the graph of $f(x) = 3^x + 1$.



b) Sketch the graph of $f^{-1}(x)$.



Question 35 — 2 marks — 128

Determine the *x*-intercept and *y*-intercept of $y = \log_2(x+4) - 1$.

Question 36 — 1 mark — 129

Explain the error that was made when solving the following equation:

$\sin 2\theta = \cos\theta$, where $\theta \in$ ¡

$\sin 2\theta = \cos\theta$

$2\sin\theta\cos\theta = \cos\theta$

$\frac{2\sin\theta\cos\theta}{\cos\theta} = \frac{\cos\theta}{\cos\theta}$

$2\sin\theta = 1$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6} + 2k\pi, \frac{5\pi}{6} + 2k\pi, k \in I$

Question 37

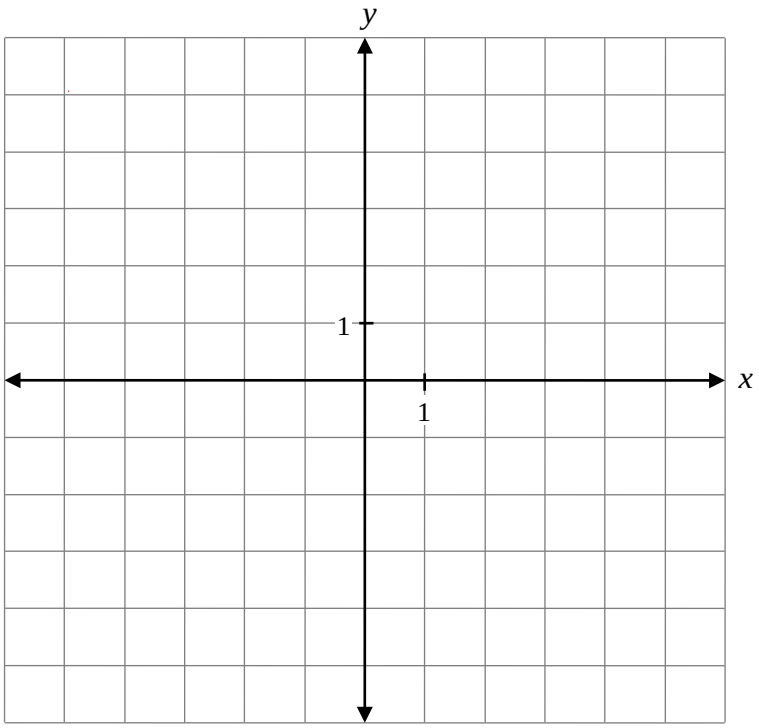
a) 1 mark b) 1 mark

130
131

Given $f(x) = x^2 - 2x - 3$ and $g(x) = x + 1$:

a) Write the equation of $y = f(g(x))$.

b) Sketch the graph of $y = f(g(x))$.



Question 38 — 1 mark — 132

Is the point $\left(\frac{3}{4}, -\frac{\sqrt{3}}{4}\right)$ on the unit circle?

Justify your answer.

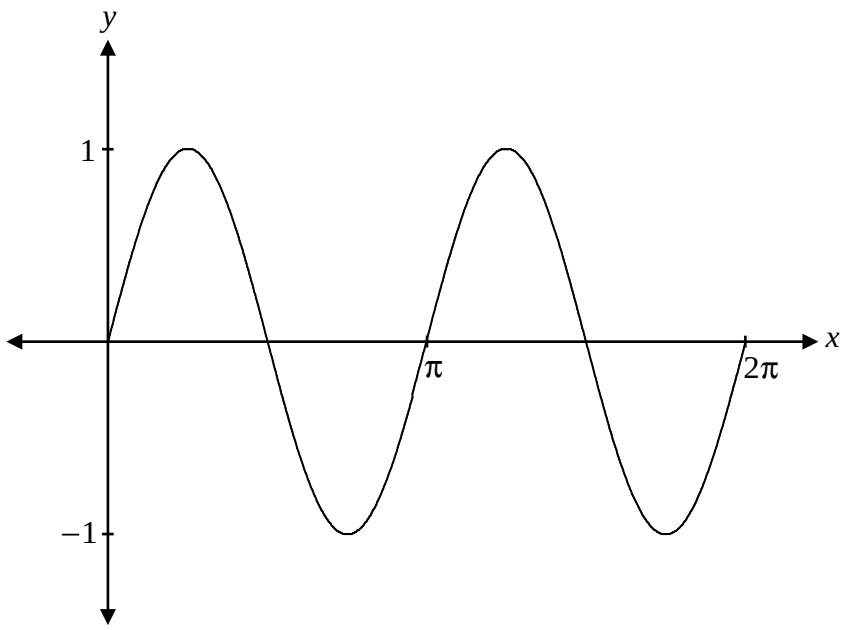
Question 39 — 1 mark — 133

Explain why the equation $\sec\theta = \frac{1}{4}$ has no solution.

Question 40 — 1 mark — 134

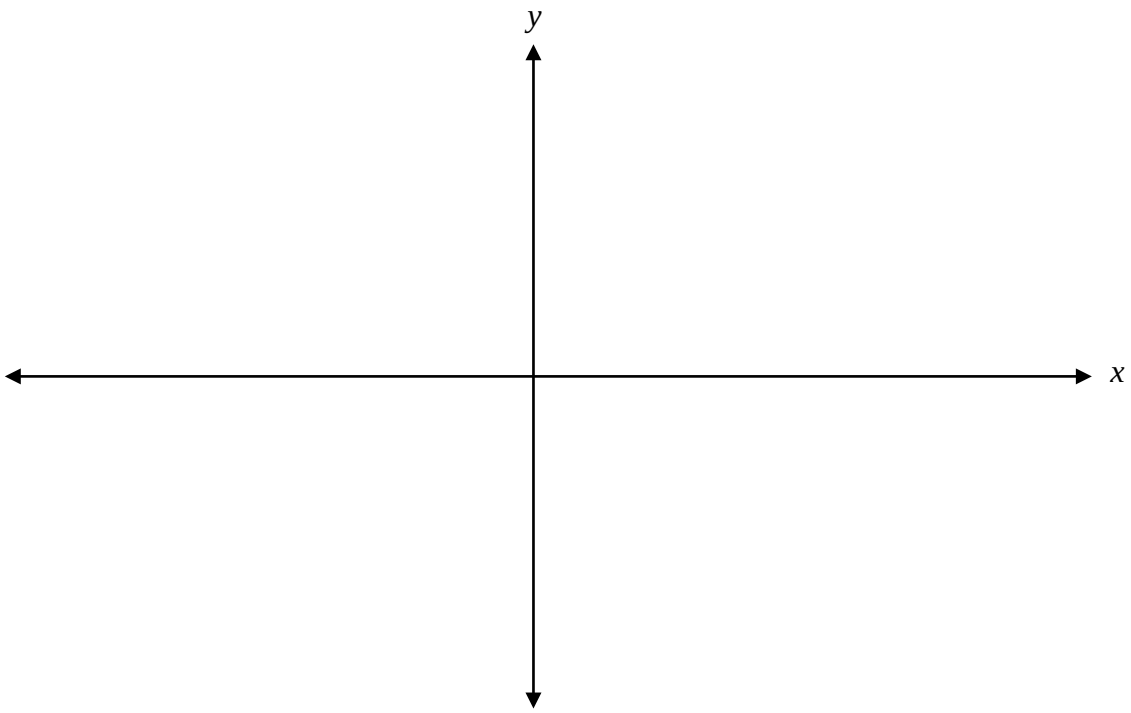
The graph of $y = \sin 2x$ is sketched below.

Explain how to use this graph to solve the equation $\sin 2x = \frac{1}{2}$ over the interval $[0,\ 2\pi]$.



Question 41 — 3 marks — 135

Sketch the graph of $y = -4\cos(2x)$ over the interval $[-\pi, \pi]$.



Question 42 — 2 marks — 136

Write the equation for $f(x)$ that satisfies all of the following conditions:

- $f(x)$ is a polynomial function of degree 4
- $f(x)$ has a zero at 2 with a multiplicity of 3
- $f(x)$ has a zero at -5
- $f(x)$ has a y-intercept of 80

Question 43 — 3 marks — 137

Find the exact value of $\sin\left(\frac{19\pi}{12}\right)$.

Question 44 — 4 marks — 138

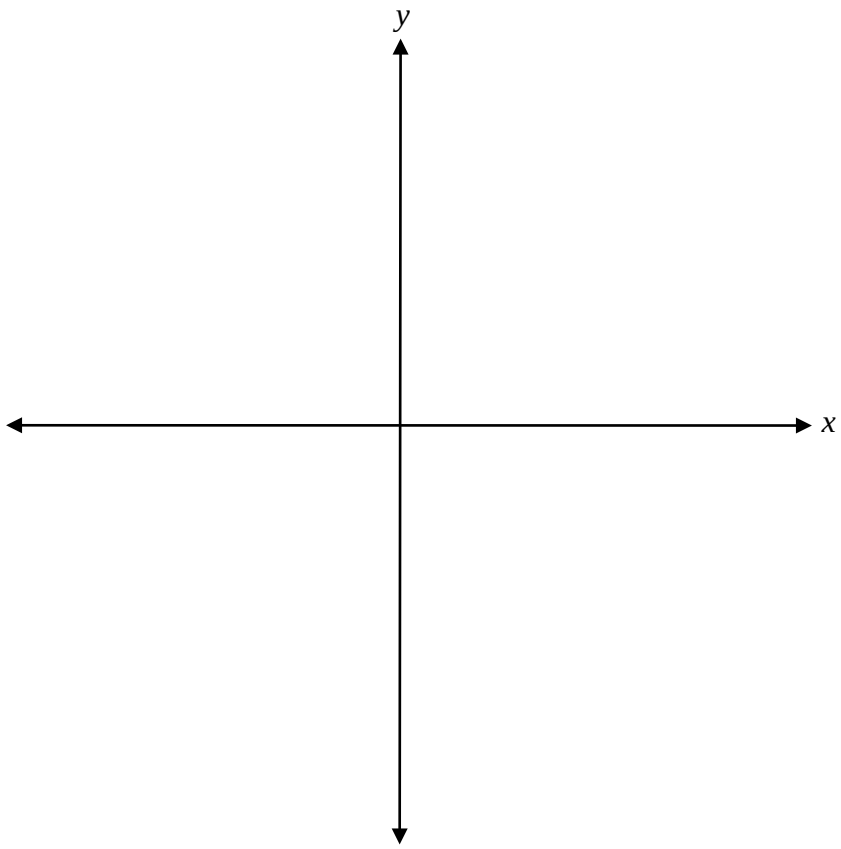
Solve the following equation:

$$2\log_2(x-1) - \log_2(x-5) = \log_2(x+1)$$

Question 45 — 3 marks — 139

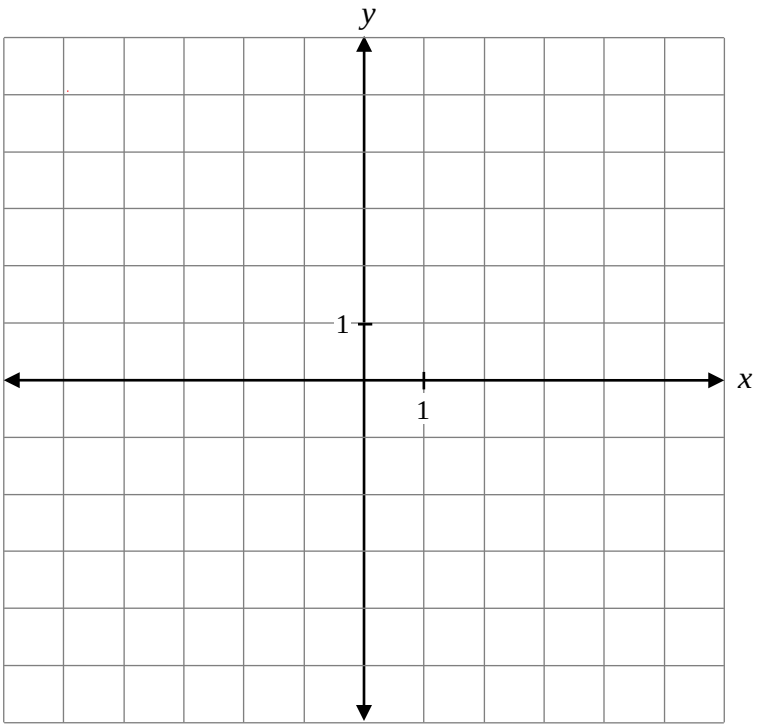
Sketch the graph of $f(x) = (x-1)^2(x+2)^3$.

Label the *x*-intercepts and the *y*-intercept.



Question 46 — 4 marks — 140

Sketch the graph of $y = -\sqrt{3(x+1)}$.



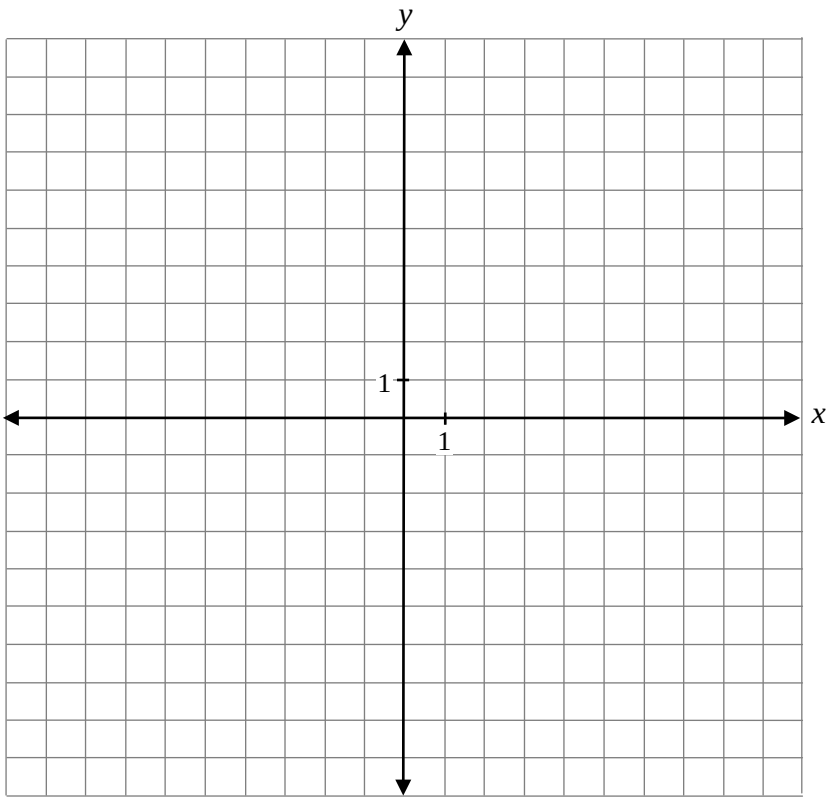
Question 47 — 3 marks — 141

Solve:

$${}_{n-1}P_2 = 42$$

Question 48 — 3 marks — 142

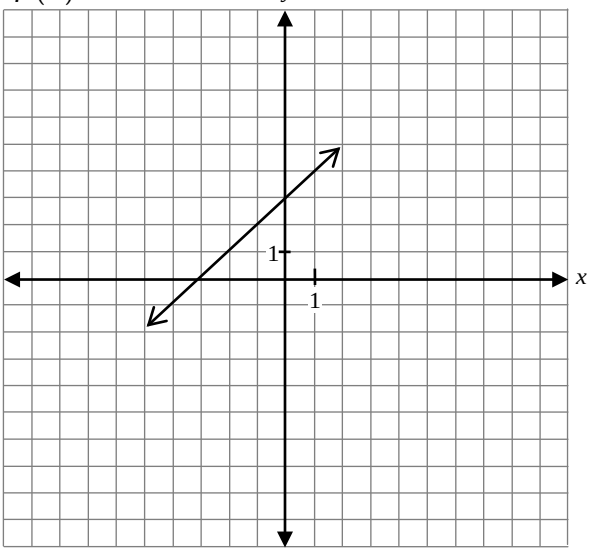
Sketch the graph of $y = \dfrac{2x}{x+2}$.



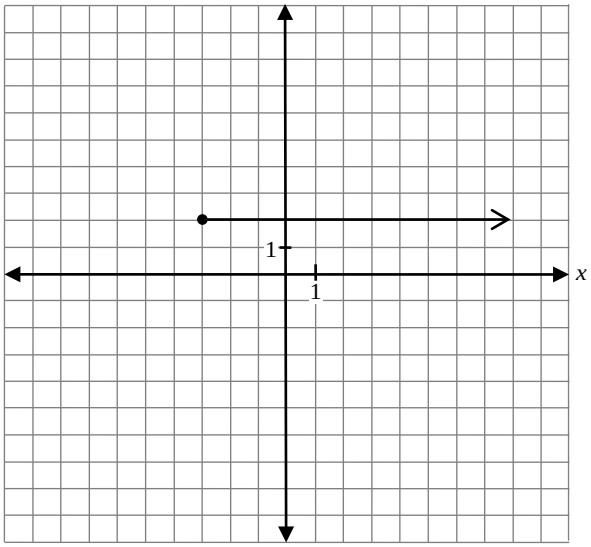
Question 49 — 2 marks — 143

Given the graphs of $f(x)$ and $g(x)$, sketch the graph of $(f \circ g)(x)$.

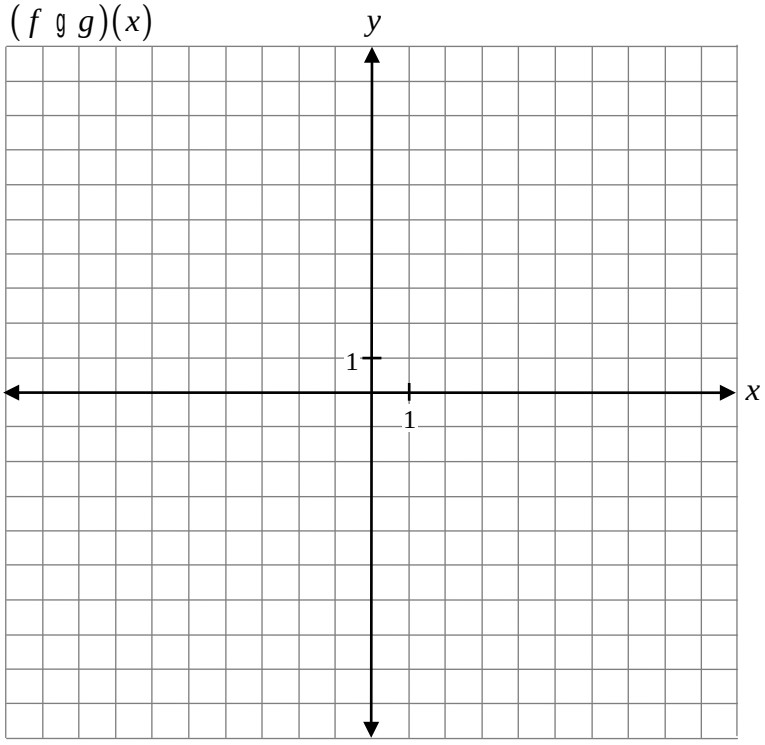
$f(x)$



$g(x)$



$(f \circ g)(x)$



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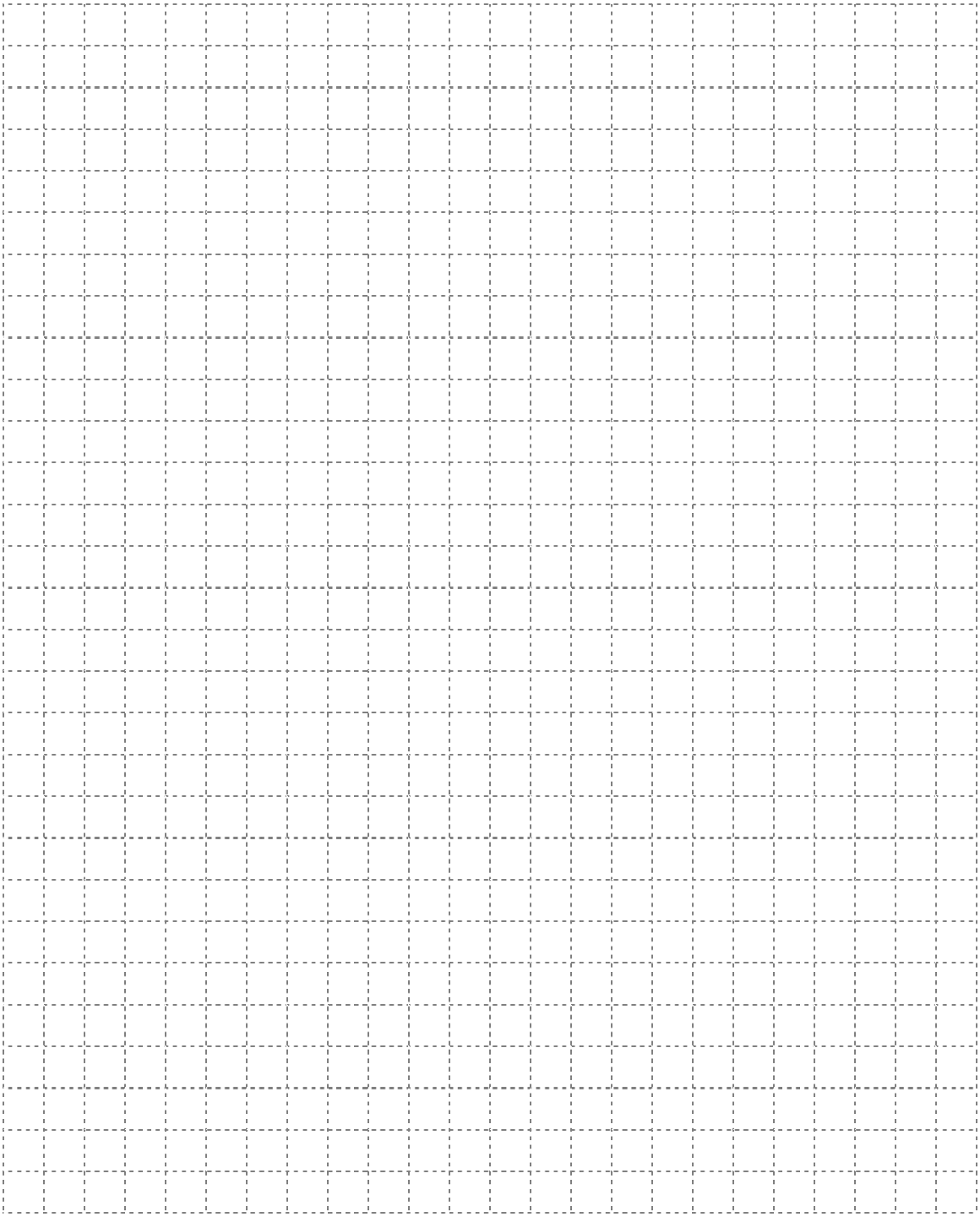

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